

ANALYTICAL BASE TRANSIT TIME MODEL OF UNIFORMLY DOPED BASE BIPOLAR TRANSISTORS CONSIDERING KIRK EFFECT

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ABSTRACT

In this paper analytical expressions of effective base width and base transit time for uniformly doped base of bipolar junction transistors (BJT) are developed taking Kirk effect into consideration. Modern bipolar junction transistors tend to operate with saturated carrier velocity in the collector space-charge region, and therefore we have incorporated this effect in the present work. Physical analysis of Kirk effect shows that the base transit time increases significantly when the base width widening or when Kirk effect is considered.

Keywords : BJT, Base Transit Time, Effective Base Width, Kirk Effect

1.0 INTRODUCTION

Modeling of collector current density and base transit time is essential for design of high-speed bipolar transistors. Some papers on collector and base transit times of a BJT have been published [1-4]. Kroemer [5] developed a base transit time model for an arbitrary doped base, and the model was improved by considering electron saturation velocity at the base-collector junction [6]. Wijnan and Gardner [7] showed that a uniform base profile gives a higher cut-off frequency than a graded profile for a given base resistance and peak base doping. In all the papers width of the base is assumed to be constant. This is a reasonable approximation when the collector current is small. At high current and high collector-emitter voltage, the injected electrons traversing the collector depletion region may become comparable to the space-charge density. Thus, the collector junction and the base width are modified [8]. This phenomenon is called the Kirk effect [9]. In the present work, base transit time for uniformly doped base will be obtained considering Kirk effect.

2.0 THEORY

The one-dimensional view of an n+pn- transistor operating in active mode is shown in Figure 1. Commonly used approximation in the calculation of base transit time is that the carrier concentration is zero at the base side of the collector-base depletion layer and effective base width is a

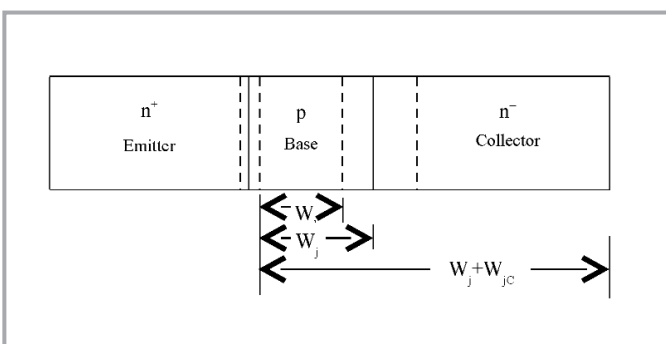


Figure 1: One dimensional view of an n+pn- bipolar transistor

constant quantity. But with carrier-velocity saturation, the carrier concentration in the space-charge region is proportional to the collector current and the position of space-charge boundary becomes function of the carrier concentration [8]. So the collector junction and the base width are modified. To explain this effect three operating conditions are considered:

- Case 1: Part of the collector region is quasi-neutral.
- Case 2: Collector is entirely space-charged.
- Case 3: Nonohmic quasi-saturation.

2.A. Effective Base Width for Case 1

The current crossing the collector junction is given by

$$J_n = qn_c v(x) \quad (1)$$

where J_n is the collector current density, $v(x)$ is the velocity of electron and n_c is the electron concentration in the space-charge region required to support the current.

Inside the depletion layer, the electric field is high so that the velocity may be assumed as having reached the saturation velocity v_s . Therefore, from equation (1) we have

$$n_c = \frac{J_n}{qv_s} \quad (2)$$

The relationship between the charge distribution and the electric field is given by the Poisson's equation as

$$\frac{d\xi}{dx} = -\frac{q}{\epsilon_s} [(n-p) + (N_a - N_d)] \quad (3)$$

where ξ is the electric field, n is the electron concentration, p is the hole concentration, N_a is the acceptor impurity concentration, N_d is the donor impurity concentration and ϵ_s is the relative permittivity or dielectric constant of the semiconductor.

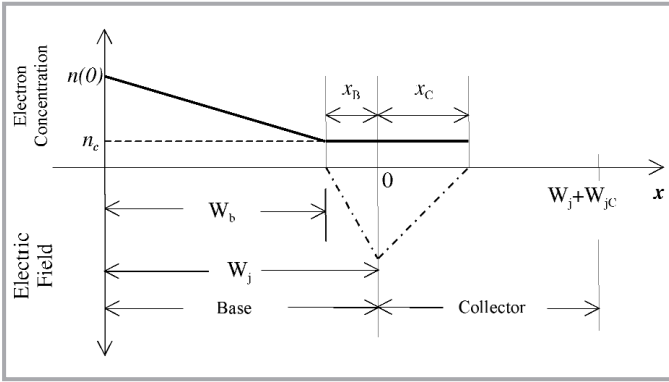


Figure 2: Typical electron concentration and electric field distribution for case 1

The electron distribution and electric field within the depletion region for case 1 is shown in Figure 2. Poisson's equation in the base side of the space-charge region is

$$-\frac{q}{\epsilon_s} \left(N_B + \frac{J_n}{qv_s} \right); \quad \text{for } -x_B < x < 0 \quad (4)$$

And in the collector side of the space-charge region is

$$\frac{d\xi_c}{dx} = \frac{q}{\epsilon_s} \left(N_C - \frac{J_n}{qv_s} \right); \quad \text{for } 0 < x < x_C \quad (5)$$

where x_B is the penetration of the space-charge region into the base, x_C is the penetration of the space-charge region into the collector, N_B is the base doping concentration and N_C is the collector doping concentration.

Since the electric field is zero in the neutral regions and at the edges of the depletion layer, we have

$$\xi_b(-x_B) = \xi_c(x_C) = 0$$

Now, integrating equations (4) and (5) and using leads to the relation

$$x_B \left(qN_B + \frac{J_n}{v_s} \right) = x_C \left(qN_C - \frac{J_n}{v_s} \right) \quad (6)$$

Poisson's equation can be solved in the collector junction space-charge region to yield the following equation:

$$V_{CB} + \phi_c = \frac{1}{2\epsilon_s} \left[\left(qN_B + \frac{J_n}{v_s} \right) x_B^2 + \left(qN_C - \frac{J_n}{v_s} \right) x_C^2 \right] \quad (7)$$

where V_{CB} is the collector junction reverse bias voltage and ϕ_c is the collector junction built-in potential.

From equations (6) and (7), x_B and x_C can be written as

$$x_B = \sqrt{\frac{2\epsilon_s N_C (V_{CB} + \phi_c)}{qN_B (N_C + N_B)}} \sqrt{\frac{1 - J_n/J_C}{1 + J_n/J_B}} \quad (8)$$

$$= x_{B0} \sqrt{\frac{1 - J_n/J_C}{1 + J_n/J_B}}$$

and

$$x_C = \sqrt{\frac{2\epsilon_s N_B (V_{CB} + \phi_c)}{qN_C (N_C + N_B)}} \sqrt{\frac{1 + J_n/J_B}{1 - J_n/J_C}} \quad (9)$$

$$= x_{C0} \sqrt{\frac{1 + J_n/J_B}{1 - J_n/J_C}}$$

Where

$$x_{B0} = \sqrt{\frac{2\epsilon_s N_C (V_{CB} + \phi_c)}{qN_B (N_C + N_B)}}, \quad x_{C0} = \sqrt{\frac{2\epsilon_s N_B (V_{CB} + \phi_c)}{qN_C (N_C + N_B)}}$$

$$J_B = qN_B v_s \quad \text{and} \quad J_C = qN_C v_s$$

So the effective base width W_b can be written as

$$W_b = W_j - x_B \quad (10)$$

where W_j is the physical base width. Equation (10) is valid only up to the value of J_n for which the base-collector space-charge region extends all the way to the collector for a given value of V_{CB} , i.e., when $x_C = W_{jc}$, W_{jc} is the physical collector width.

The upper limit of validity for equation (10) can be found from equation (9) with x_C replaced by W_{jc}

$$J_n|_{x_C=W_{jc}} \cong \frac{J_C}{1 + \frac{N_B}{N_C} \left(\frac{x_{B0}}{W_{jc}} \right)^2} \left[1 - \left(\frac{x_{C0}}{W_{jc}} \right)^2 \right] \quad (11)$$

2.B. Effective Base Width for Case 2

As J_n increases above $J_n|_{x_C=W_{jc}}$, the collector-base space-charge region begins to uncover charge in the buried layer. The resulting electron concentration and electric field distribution are shown in Figure 3(a). As the current increases above $J_C (= qN_C v_s)$, the electron concentration in the collector

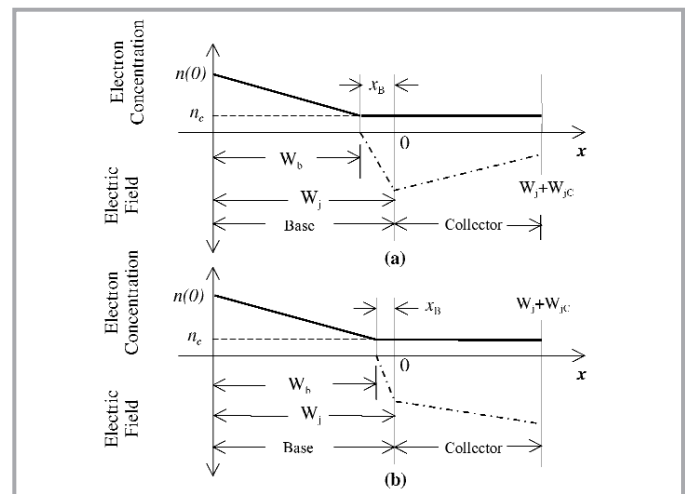


Figure 3: Typical electron concentration and electric field distribution for case 2 (a) for $J_n < J_C$, (b) for $J_n > J_C$

risers above N_c , leading to electric field and electron concentration distributions as shown in Figure 3(b).

In this case the electric field distribution in the base and collector region will be

$$\xi_b(x) = -\frac{q}{\epsilon_s} \left(N_B + \frac{J_n}{qv_s} \right) (x + x_B) \quad (12)$$

and

$$\xi_c(x) = \xi_c(W_{jC}) - \frac{q}{\epsilon_s} \left(N_c - \frac{J_n}{qv_s} \right) (W_{jC} - x) \quad (13)$$

where

$$\xi_c(W_{jC}) = \frac{1}{\epsilon_s v_s} [(J_c - J_n)W_{jC} - (J_B + J_n)x_B]$$

Using charge neutrality and solving Poisson's equation over the collector junction space-charge region, x_B for case 2 can be written as

$$x_B \cong -W_{jC} + \sqrt{W_{jC}^2 \left(1 + \frac{N_c(1 - J_n/J_c)}{N_B(1 + J_n/J_B)} \right) + \frac{2\epsilon_s(V_{CB} + \phi_c)}{qN_B(1 + J_n/J_B)}} \quad (14)$$

The effective base width W_b for case 2 can be written as

$$W_b = W_j - x_B \quad (15)$$

2.C. Effective Base Width for Case 3

Further increment in J_n drives the BJT into nonohmic quasi-saturation. Figure 4 shows a typical electron concentration and electric field distribution in nonohmic quasi-saturation condition. In this condition of operation, the collector space-charge region excess free-electron concentration ($n_c - N_c$) is balanced by the heavily doped buried-layer charge.

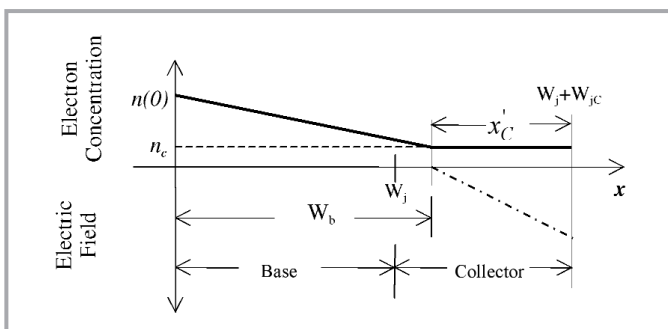


Figure 4: Typical electron concentration and electric field distribution for case 3

$$x'_C = \sqrt{\frac{2(V_{CB} + \phi_c)\epsilon_s v_s}{J_n - J_c}} \quad (16)$$

where x'_C is the collector space-charge width in nonohmic quasi-saturation operation. In this case W_b must be redefined as

$$W_b = W_j + W_{jC} - x'_C \quad (17)$$

The upper limit of validity for equation (15) and lower limit for equation (17) can be found by replacing x_E with W_{jC} in equation (16) as

$$J_n \Big|_{x'_C=W_{jC}} = \frac{2\epsilon_s v_s (V_{CB} + \phi_c)}{W_{jC}^2} + J_c \quad (18)$$

2.D. Collector Current Density

Neglecting the recombination within the base the electron current density, J_n of a bipolar transistor can be written as

$$J_n = qD_n \frac{dn(x)}{dx} + q\mu_n \xi(x)n(x) \quad (19)$$

where q is the electron charge, D_n is the electron diffusion coefficient, μ_n is the electron mobility, n is the electron concentration and $\xi(x)$ is the electric field given by [10]:

$$\xi(x) = \frac{kT}{q} \frac{1}{N_B + n(x)} \frac{dn(x)}{dx} \quad (20)$$

The carrier recombination in the base region is negligible in today's bipolar transistors [6], which makes J_n constant. Integrating equation (19) within the base, we obtain:

$$-\frac{J_n}{qD_n} W_b = 2[n(W_b) - n(0)] - N_B \ln \left[\frac{N_B + n(W_b)}{N_B + n(0)} \right] \quad (21)$$

where $n(0)$ is the electron concentration in the base at the edge of emitter-base space-charge region and $n(W_b)$ is the electron concentration in the base at the edge of collector-base space-charge region.

The boundary condition at the emitter-base junction is given by [11]

$$n(0) = \frac{n_{ie}^2}{N_B} \exp \left(\frac{qV_{BE}}{kT} \right) \quad (22)$$

and, assuming that the electron velocity in the base-collector depletion region saturates at v_s , $n(W_b)$ can be expressed as [12]

$$n(W_b) = \frac{J_n}{qv_s} \quad (23)$$

Since $n(W_b)$ is much smaller than N_B , we can then expand the last term in equation (21) into a Taylor series, which leads to the following analytical expression for collector current density:

$$J_n = \frac{qn(0)}{\frac{W_b}{D_n} + \frac{1}{v_s}} \left[2 - \frac{N_B}{n(0)} \ln \left(1 + \frac{n(0)}{N_B} \right) \right] \quad (24)$$

2.E. Base Transit Time

For a uniform base profile, the minority carrier concentration in the base for a thin base bipolar transistor can be written by the well-known expression [2]

$$n(x) = n(0) - \frac{[n(0) - n(W_b)]x}{W_b} \quad (25)$$

Integrating the injected minority carrier over the effective base region and dividing it by J_n/q [5], base transit time τ_B can be solved as

$$\tau_B = \frac{\frac{W_b^2}{2D_n} + \frac{W_b}{v_s} \left[\frac{3}{2} - \frac{N_B}{2n(0)} \ln \left(1 + \frac{n(0)}{N_B} \right) \right]}{2 - \frac{N_B}{n(0)} \ln \left(1 + \frac{n(0)}{N_B} \right)} \quad (26)$$

3.0 RESULTS

In modern bipolar junction transistors the collector is more lightly doped than the base. Therefore high values of collector current and collector-emitter voltage can reasonably change the effective base width. From the equations (10), (15) and (17) it is clear that the effective base width is not constant - it is a function of collector current density J_n . The dependence of effective base width (W_b) with collector

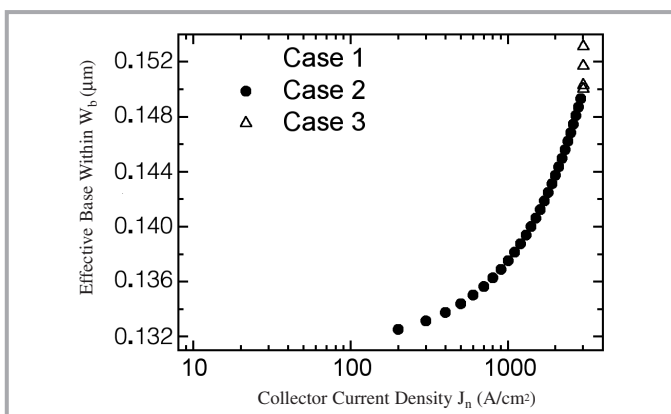


Figure 5: Effective base width as a function of collector current density for case 1, 2 and 3. Here $N_B = 1 \times 10^{18} \text{ cm}^{-3}$ and $W_j = 0.15 \text{ } \mu\text{m}$

current density (J_n) is shown in Figure 5. From the figure it is clear that the effective base width is almost constant in case 1. But in case 2 effective base width increases rapidly with the increase of collector current density J_n . This is because when the collector is fully depleted, increase in J_n decreases the depletion layer in the base side rapidly. In case 3 the base spreads over the lightly doped collector region and the effective base width becomes very high.

The variation of base transit time with base-emitter voltage for different base doping concentrations is shown in Figures 6 and 7. Figure 6 is for case 1 and case 2, and Figure 7 is for case 3. It is clear from the figures that with the

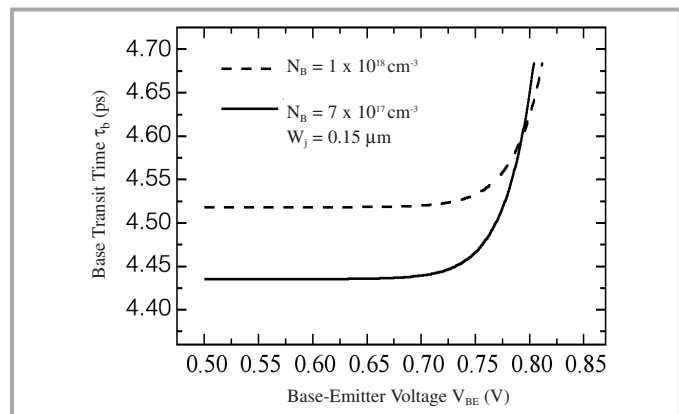


Figure 6: Base transit time as a function of base-emitter voltage for different values of N_B . Here $W_j = 0.15 \text{ } \mu\text{m}$ (for case 1 and 2)

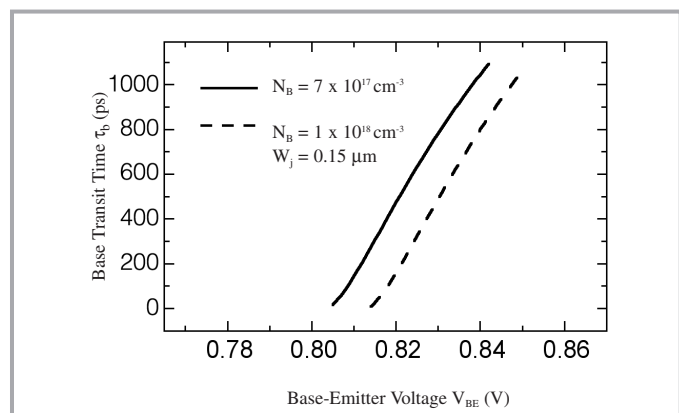


Figure 7: Base transit time as a function of base-emitter voltage for different values of N_B . Here $W_j = 0.15 \text{ } \mu\text{m}$ (for case 3)

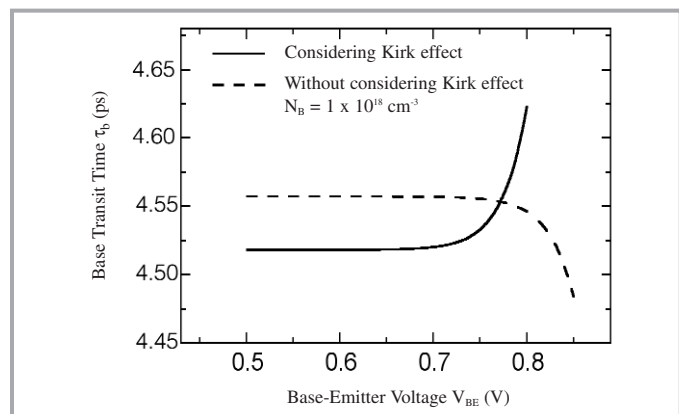


Figure 8: Comparison of base transit time with and without considering Kirk effect for $N_B = 1 \times 10^{18} \text{ cm}^{-3}$

increase of base-emitter voltage, base transit time increases. With the increase of base-emitter voltage the collector current increases, which can reduce the base transit time. But at the same time effective base width increases with the increase in base-emitter voltage. Since with the increase of effective base width electrons injected from the emitter have to traverse more distance, the base transit time increases.

In Figure 8, base transit time with and without considering Kirk effect is shown. From this figure we see that base transit time without considering Kirk effect is almost constant for lower values of V_{BE} and decreases for higher values of V_{BE} . But considering Kirk effect we see that actually base transit time increases with base-emitter voltage V_{BE} .

4.0 CONCLUSION

The results show that base transit time increases with base-emitter voltage. The results obtained are compared with the results in which Kirk effect was neglected. In modern high-speed bipolar junction transistors the collector is more lightly doped than the base resulting the base widening. So neglecting the Kirk effect we will not get the actual value of base transit time which will affect the design the different parameters like maximum frequency of operation, cut-off frequency and noise figure of modern high-speed BJTs. ■

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PROFILE



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