2-CYCLE MOMENT DISTRIBUTION FOR THE ANALYSIS OF CONTINUOUS BEAMS AND MULTI-STOREY FRAMED STRUCTURES

Chan Kam Leong

B.Sc(Eng.), MSc, P.Eng, C.Eng, MICE, MIStruct E, MIEM, MACEM K.L. Chan & Associates, Consulting Engineers

E-mail: klchanassociates@yahoo.com

ABSTRACT

In the 2-Cycle Moment Distribution, moments are distributed twice regardless of the number of spans. Moments are carried over first and are included with fixed-end moments before the distribution is made. Both dead load and total load are distributed simultaneously to obtain critical moments at supports as well as spans. Besides, maximum moments in the columns can be obtained fairly quickly.

Keywords: *Adjustments, Balancing Moments, Carry-Over, Distribution Factor, Fixed-End Moment, Maximum Moment, Midspan Moment, Minimum Moment, Support Moment*

INTRODUCTION

Rigid frame and continuous beam analysis is a very important subject for the structural engineer.

The method of moment distribution was first introduced by Professor Hardy Cross to his students at the University of Illinois (USA) in 1924 and published in 1929 [1]. It has been considered one of the greatest contributions ever made to structural theory. The method has provided a means to analyse many types of continuous frames, which were formerly designed by empirical rules or approximate methods, can be analysed with accuracy and comparative ease.

Thirty years later, the Portland Cement Association of the USA published the 2-Cycle Moment Distribution Method in 1959 [1]. This is not a new method and it has been tested over a period of years in the analysis of numerous building frames. Its speed and accuracy certainly are of great assistance to designers.

The author came to know about this method when he was working for a consulting firm in London in 1972. He wrote an article on the 2-Cycle Method which was published in the IEM Bulletin in August 1978 [2] and Dr. Leong Tuck Wah of the University of Singapore also wrote and published in the IEM Bulletin in December 1978 [3].

It is considered necessary to revise the article. However, the readers are assumed that they are familiar with the Hardy Cross moment distribution method.

As the name implies, the 2-Cycle Moment Distribution distributes moments twice regardless of the number of spans in a continuous frame. Moments are carried over first and are included with fixed-end moments before the distribution is made. And both dead load (D.L.) and total load (T.L.) are distributed simultaneously to obtain critical moments at supports as well as spans. Besides, maximum moments in the columns can be obtained fairly quickly. Whereas in the Cross Method the process of distribution is repeated for many cycles in order to bring balancing or carry-over moments to very small magnitudes. It can only obtain moments at supports but not at spans. Besides, dead load and total load are distributed separately. The Cross Method is comparatively time consuming.

THE CONCEPT OF FIXED-END MOMENT

In Figure 1(a), beam AB deflects under a load P and the tangents at the ends will rotate through angles θ_A and θ_B.

In Figure 1(b), the end at A is restrained by a moment M_{AB} and the angle change at A is smaller than θ_A.

In Figure 1(c), when the angle changes are zero at both supports A and B, the beam AB is said to have fixed ends, and the restraining moments are called fixed-end moments, M^F_{AB} and M^F_{BA}.

The fixed-end moment is very useful in beam design since it is independent of other members in the frame and also is a major part of the actual end moment in the beam. One objective in frame analysis is to determine the minor correction to the fixed-end moment to give the actual moment.

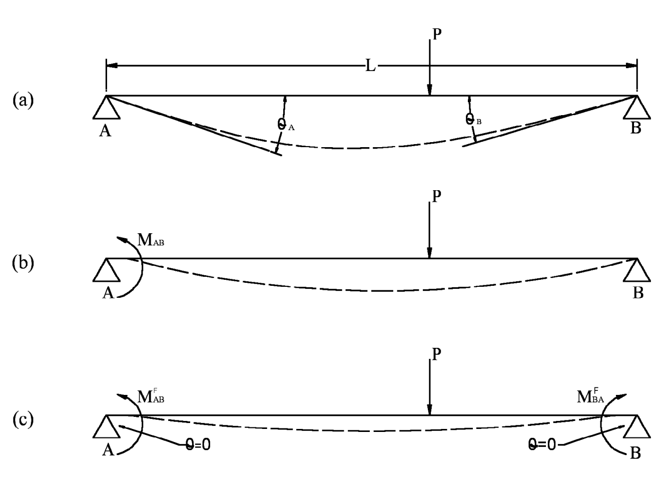


Figure 1: Beam with various degrees of restraint

STIFFNESS AND CARRY OVER FACTOR

In Figure 2, beam AB of constant cross section is simply supported at B and fixed at A. A counter-clockwise rotation of θ_B may be effected by applying a counter-clockwise moment of M_{BA} at B, and this in turn induces a resisting moment M_{AB} on the member at A.

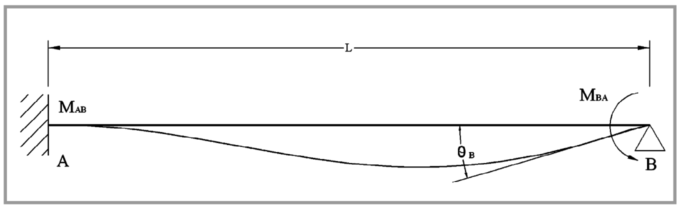


Figure 2: Moments in beam with one end fixed, other end being rotated

It can be proved that $M_{BA} = K\theta_B$ and $M_{AB} = \frac{1}{2} M_{BA}$, where K is the stiffness of the member. For the members with constant section, K equals $\frac{4EI}{L}$, which is referred to as the absolute value. A relative value of K = $\frac{I}{L}$ is preferred when E is constant throughout a frame, where E is the modulus of elasticity, I is the second moment of inertia and L is the member length. The two equations show that:

1. The stiffness K at B equals the moment at B required to give B a unit rotation when A is fixed.
2. The moment required to rotate B through a given angle is proportional to the stiffness K.
3. Applying a moment M_{BA} at B will induce at A a moment $M_{AB} = \frac{1}{2} M_{BA}$. The factor of $\frac{1}{2}$ is called the "carry-over factor".

The concepts of stiffness and carry-over factor together with the concept of fixed-end moment are used in moment distribution.

SIGN CONVENTIONS

Fixed-end moments are considered negative on both sides of a joint. Hence, moments are negative in "humps" (tension in top) and positive in "sags" (tension in bottom).

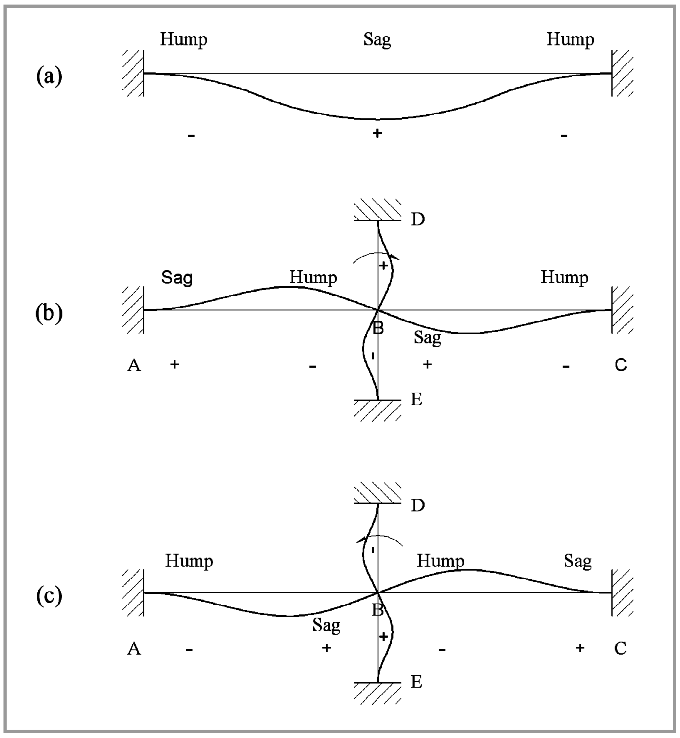


Figure 3: Signs illustrated by means of curvature and deflection of beams and columns

In Figure 3(a), the fixed-ended beam has central portion sags (plus) and the outer portions hump (minus).

In Figures 3(b) and 3(c), the clockwise and counter-clockwise rotation about a central support B, of a continuous, fix-ended frame show that the beam sags on one side and humps on the other side. It is also clear that the beam sags at one end of a member because of joint rotation, it will hump at the opposite end. Similarly, under the action of a moment, a column can be treated as a beam when viewed from the right and the same sign convention for beams can be applied.

MOMENT DISTRIBUTION AT ONE JOINT

In Figure 4(a), the frame consists of four members fixed at their far ends. The moment U will rotate at joint B until the sum of the resisting moments induced in the four members is equal to U. Since all members are rigidly connected at B, each member will rotate through the same angle at this joint.

In Figures 4(b) and 4(c), joint B is being rotated clockwise by an external moment U = 69kNm. Stiffnesses, distribution factors, distributed moments and carry-over moments are as shown.

Figure 4(a) indicates that the clockwise rotation of joint B creates a hump to the left, but a sag to the right. Therefore 19 is negative but 18 is positive. There is also a sag at A and a hump at D; therefore the carried-over moments are +10 at A and -9 at D. In moment distribution, U is called the "unbalanced moment" and is computed as the numerical difference between adjacent fixed-end moments.

Viewing from the right, the vertical members (which become horizontal), Figure 4(a) indicates that the clockwise rotation of joint B creates a hump to the left, but a sag to the right of B. Therefore 22 is negative and 10 is positive. There is also a sag at E and a hump at C; therefore the carried-over moments are +11 at E and -5 at C.

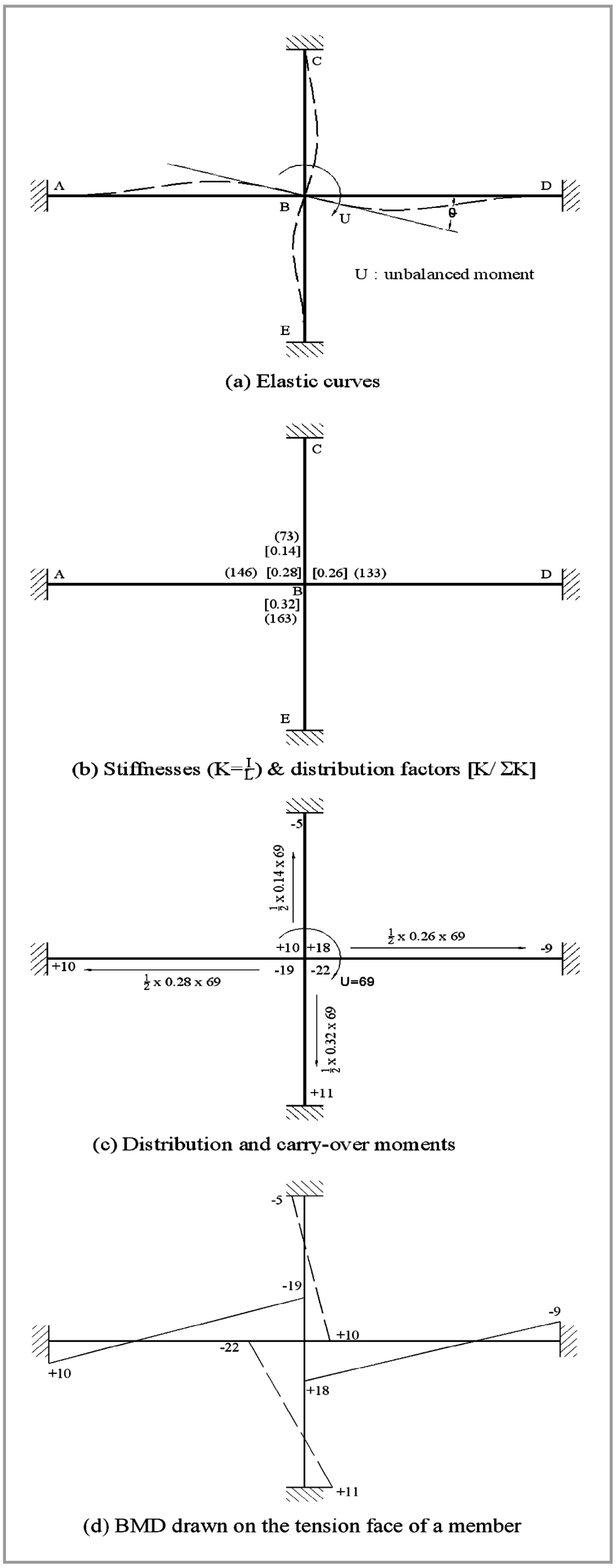
Figure 4(d) shows a BMD drawn on the tension face of a member.

MAXIMUM SUPPORT MOMENT

As the name implies, the 2-Cycle Moment Distribution only 'distributes' twice regardless of the number of spans in a continuous beam or frame. It also distributes total load and dead load simultaneously. This can be best illustrated by an example.

Let us consider the example of a 4-span continuous beam as shown in Figure 5. The problem is to determine maximum support moments. Figure 6 contains five groups of calculations for moments at the supports.

To assume maximum moment at support A, place total load on span AB and dead load on span BC as shown in Figure 6. As long as support B is considered fixed, the end moments at B are 67.5 to the left and 20 to the right. The difference is 47.5. When B is released, the moment distributed to the left = 47.5 x 2/5, and the moment carried over to A while A remains fixed is 47.5 x 2/5 x $\frac{1}{2}$ = 9.5. The counter-clockwise rotation of joint B creates a hump in the beam at A. Therefore the moment of 9.5 carried over to A is negative. Joint B is then relocked in its new position. Now turn to A which so far has been considered locked. The original fixed-end moment is -67.5, but the release and rotation of B transfers an additional moment to A, and at this stage the modified total fixed-end moment is -67.5 – 9.5 = -77. Since there is no fixed-end moment to the left of A, the unbalanced moment equals 77. Releasing A and permitting it to rotate induces a distributed moment at A equals to 77 x 1 = 77. When joint A rotates clockwise



***Distributed moment:* [K/∑K] x U, *carry-over moment:*½ x [K/∑K] x U**

Figure 4: Frame consisting of four members with far ends fixed

it tends to create a sag in the beam at A. Therefore 77 is positive and the final maximum moment at A is -77+ 77 = 0.

To determine moments at B, begin with releasing joints A and C, Figure 6 shows how to compute the two moments 33.75 and 0.35 carried over to B. When A and C are released they rotate so as to create a hump on one side and a sag on the other side of the fixed joint B, therefore 33.75 is negative and 0.35 is positive. While B is still considered fixed, the modified total fixed-end moments at B are -101.25 to the left and -29.65 to the right and out of balance at B is -101.25 + 29.65 = -71.6. Release joint B, multiply 71.6 by the distribution factors 2/5 and 3/5, and record the results 28.64 and 42.96 as shown. The counter clockwise rotation at joint B creates a sag to the left (+28.64) and a hump to the right (-42.96). Distributed or balancing moments at joints C, D and E are determined by the same procedure as shown in Figure 6.

The operations illustrated in Figure 6 show that moments are distributed twice. Moments are carried over first and are included with fixed-end moments before the distribution is made. In the previous calculations, the five groups have five different arrangements of loads i.e. total loads on spans adjacent to the particular joint at which maximum moments are computed but dead loads only on the next outside spans. Yet the calculations as shown in Figure 6 can be consolidated into one single group without any interference as shown in Figure 5. It is clear that the results as shown in Figure 6 between the 2-Cycle Moment Distribution and computer analysis are quite close.

THE METHOD OF CALCULATING MAXIMUM SUPPORT MOMENT:

Step 1 Write down D.F., MD.L & MT.L. ,

where $M = -\dfrac{WL^2}{12}$ for a uniformly distributed load, W.

Step 2 Calculate and write down C/O
$C/O = \frac{1}{2} \times D.F \times (M_{T.L} - M_{D.L})$

Step 3 Obtain total unbalanced moment ($\sum M$) by adding $M_{T.L}$ to C/O

Step 4 Balancing moments are of opposite sign to reduce larger moment.

Step 5 Add $\sum M$ to balancing moment for max. support moment.

MAXIMUM MOMENT AT MIDSPAN

To determine maximum moments at midspan the positive midspan moments shown as +33.75, +15.0, + 23.44 and +33.75 are taken from Figure 5 for beams with fixed ends. Certain corrections are to be added to these moments in order to obtain the final maximum moments at midspan.

The procedure is illustrated for span BC in Figure 7.
Distribution = D.F ($M_{T.L} - M_{D.L}$).

The distributed moments induced at B and C give 9 and 0.69 respectively.

The counter-clockwise rotation of moment at B creates a hump to the right of B and a sag to the left of C. Therefore, 9 is negative and 4.5 is positive. The clockwise rotation of moment at C creates a hump to the left of C and a sag to the right of B. Therefore 0.69 is negative and 0.35 is positive. The counter-clockwise rotation of 0.35 creates a hump to the right of B. Therefore 0.21 is negative. The clockwise rotation of 4.5 creates a hump to the left of C. Therefore 2.5 is negative.

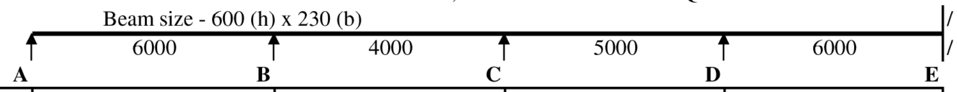
Legend

D.F. - Distribution Factor
$M_{D.L.}$ - Fixed End Moment due to Dead Load
$M_{T.L.}$ - Fixed End Moment due to Total Load
C/O - Carry Over Moment
Ratio - $M_{\text{2-cycle method}} / M_{\text{computer}}$

Sign Convention

+VE - For Sagging Moment
-VE - For Hogging Moment

D.L.=1.0G = 15.0 kN/m , T.L.= 1.4G + 1.6Q = 22.5 kN/m



	A	B	B	C	C	D	D	E
D.F.	1.0	2/5	3/5	5/9	4/9	6/11	5/11	0
$M_{D.L.}$	-45.00	-45.00	-20.00	-20.00	-31.25	-31.25	-45.00	-45.00
$M_{T.L.}$	-67.50	-67.50	-30.00	-30.00	-46.88	-46.88	-67.50	-67.50
C/O	(a) -9.50	(b) -33.75	(c) +0.35	(d) +4.50	(e) -0.52	(f) -5.97	(g) 0	(h) -8.24
ΣM	-77.00	-101.25	-29.65	-25.50	-47.40	-52.85	-67.50	-75.74
Bal. M	(i) +77.00	(j) +28.64	(k) -42.96	(l) -12.16	(m) +9.73	(n) -7.99	(o) +6.66	(p) 0
Max Supp M	0	-72.61	-72.61	-37.66	-37.66	-60.84	-60.84	-75.74
Computer	0	-75.80	-75.80	-33.80	-33.80	-63.90	-63.90	-75.50
Ratio	1.0	0.96	0.96	1.11	1.11	0.95	0.95	1.0
Midspan $M_{T.L.}$	+33.75		+15.00		+23.44		+33.75	
Adj. M_l	+9.50 (q)		-0.28 (s)		+0.38 (u)		0.00 (w)	
Adj. M_r	+23.63 (r)		-3.50 (t)		+4.61 (v)		+4.12 (x)	
Midspan M	+66.88		+11.22		+28.43		+37.87	
Computer	+69.90		+9.6		+33.50		+37.90	
Ratio	0.96		1.17		0.85		1.00	

Figure 5: Max. M (Support & Midspan) for a continuous beam over supports providing no restraint to rotation at A, B, C and D

C/O = 1/2 x D.F. x ($M_{T.L.}$ - $M_{D.L.}$)

(a) = 1/2 x 2/5 x [-67.50 - (-20.00)] = -9.5
(b) = 1/2 x 1.0 x (-67.50 - 0.00) = -33.75
(c) = 1/2 x 5/9 x [-30.00 - (-31.25)] = +0.35
(d) = 1/2 x 3/5 x [-30.00 - (-45.00)] = +4.50
(e) = 1/2 x 6/11 x [-46.88 - (-45.00)] = -0.52
(f) = 1/2 x 4/9 x [-46.88 - (-20.00)] = -5.97
(g) = 1/2 x 0 x (-75.74 - 0) = 0
(h) = 1/2 x 5/11 x [-67.50 - (-31.25)] = -8.24

Balacing Moments are of opposite sign to reduce larger moment:

(i) = - (-77.00-0.00) x 1.0 = +77.00
(j) = +(101.25-29.65) x 2/5 = +28.64
(k) = -(101.25-29.65) x 3/5 = -42.96
(l) = +(25.50-47.40) x 4/9 = -12.16
(m) = -(25.50-47.40) x 5/11 = + 9.73
(n) = +(52.85-67.50) x 6/11 = - 7.99
(o) = -(52.85-67.50) x 5/11 = + 6.66
(p) = +(75.74 - 0.00) x 0 = 0

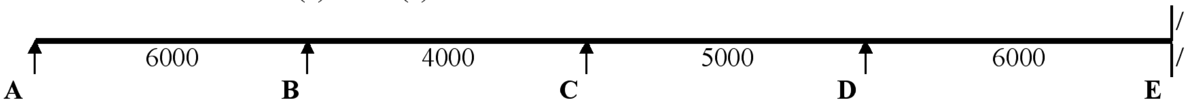
Adjustments of Span Moment due to Supports Moments:

- 1/2 x (1.0 + D.F.) x C/O

(q) = - 1/2 x (1.0 + 1.0) x (-9.50) = +9.50
(r) = - 1/2 x (1.0 + 2/5) x (-33.75) = +23.63
(s) = - 1/2 x (1.0 + 3/5) x (+0.35) = - 0.28
(t) = - 1/2 x (1.0 + 5/9) x (+4.50) = - 3.50
(u) = - 1/2 x (1.0 + 4/9) x (-0.52) = +0.38
(v) = - 1/2 x (1.0 + 6/11) x (-5.97) = +4.61
(w) = - 1/2 x (1.0 + 5/11) x (0) = 0
(x) = - 1/2 x (1.0 + 0.0) x (-8.24) = +4.12

D.L.=1.0G = 15.0 kN/m , T.L.= 1.4G + 1.6Q = 22.5 kN/m

Beam size - 600 (h) x 230 (b)



Calculate Support Moment for Joint A and E

Load Pattern	T.L.		D.L.	D.L.	T.L.	
D.F.	1.0	2/5	3/5	6/11	5/11	0.0
$M_{D.L.}$			-20.00	-31.25		
$M_{T.L.}$	-67.50	-67.50			-67.50	-67.50
C/O	(a) -9.50					(h) -8.24
ΣM	-77.00					-75.74
Bal. M	(i) +77.00					(p) 0
Max Supp M	0					-75.74
Computer	0					-75.40

Calculate Support Moment for Joint B

Load Pattern	T.L.		T.L.		D.L.
D.F.	1.0	2/5	3/5	5/9	4/9
$M_{D.L.}$					-31.25
$M_{T.L.}$	-67.50	-67.50	-30.00	-30.00	
C/O		(b) -33.75	(c) +0.35		
ΣM		-101.25	-29.65		
Bal. M		(j) +28.64	(k) -42.96		
Max Supp M		-72.61	-72.61		
Computer		-74.7	-74.7		

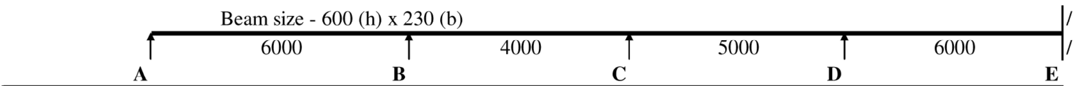
Calculate Support Moment for Joint C

Load Pattern	D.L.	T.L.		T.L.		D.L.
D.F.	2/5	3/5	5/9	4/9	6/11	5/11
$M_{D.L.}$	-45					-45
$M_{T.L.}$		-30	-30	-46.88	-46.88	
C/O			(d) +4.5	(e) -0.52		
ΣM			-25.5	-47.4		
Bal. M			(l) -12.16	(m) +9.73		
Max Supp M			-37.67	-37.67		
Computer			-34.20	-34.20		

Calculate Support Moment for Joint D

Load Pattern	D.L.	T.L.		T.L.	
D.F.		4/9	6/11	5/11	0.0
$M_{D.L.}$	-20				
$M_{T.L.}$		-46.88	-46.88	-67.5	-67.5
C/O			(f) -5.97	0	
ΣM			-52.85	-67.5	
Bal. M			(n) -7.99	(o) +6.66	
Max Supp M			-60.84	-60.84	
Computer			-62.3	-62.3	

Figure 6: Moment distribution illustrated in its various elements



	A–B	B–C	C–D	D–E
D.F.	2/5	3/5 5/9	4/9	
$M_{D.L.}$	-45	-20 -20	-31.25	
$M_{T.L.}$	-67.5	-30 +15 -30	-46.88	
Distribution		-9 -0.69		
C/O		+0.35 +4.5		
Distribution		-0.21 -2.50		

Figure 7(a)

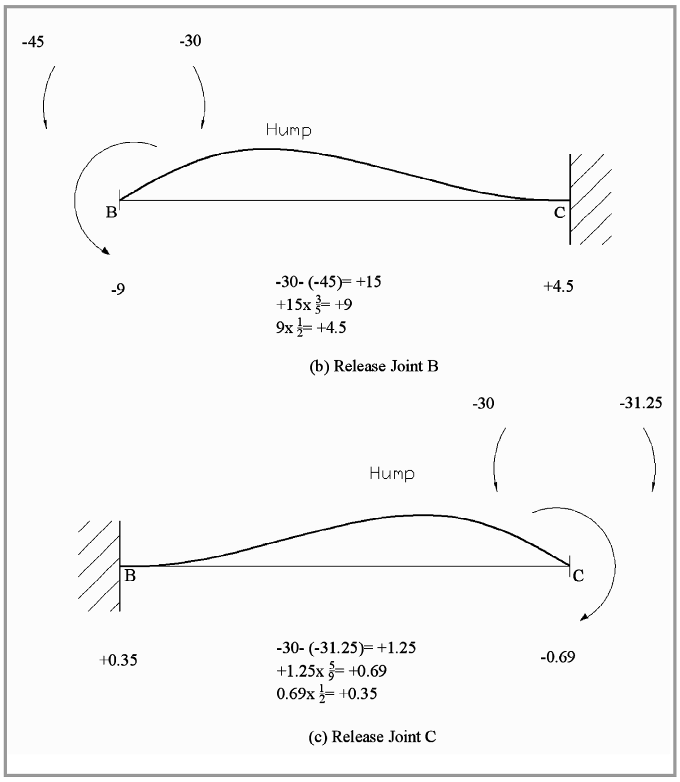


Figure 7: Two corrections for Midspan M

The corrections or adjustments at supports B and C are (-9+0.35-0.21) and (-0.69+4.5-2.5) respectively. Therefore the corrections or adjustments at the midspan are $\frac{(-9 + 0.35 - 0.21)}{2}$ and $\frac{(-0.69 + 4.5 - 2.5)}{2}$. It can be shown that the corrections or adjustments are $\left[-\frac{1}{2}(1 + D.F)\ x\ C/O\right]$ due to left hand support at B and $\left[-\frac{1}{2}(1 + D.F)\ x\ C/O\right]$ due to right hand support at C as follows:

$$\frac{-9 + 0.35 - 0.21)}{2} + \frac{-0.69 + 4.5 - 2.5}{2}$$

$$= \frac{-9 + 4.5 - 2.5}{2} + \frac{-0.69 + 0.35 - 0.21}{2}$$

$$= \frac{-9 + (9 \text{ x } 0.5) - (9 \text{ x } 0.5 \text{ x } \frac{5}{9})}{2} + \frac{-0.69 + (0.69 \text{ x } 0.5) - (0.69 \text{ x } 0.5 \text{ x } \frac{3}{5})}{2}$$

$$= \frac{9 \text{ x } 0.5}{2}\left(-\frac{1}{0.5} + 1 - \frac{5}{9}\right) + \frac{0.69 \text{ x } 5}{2}\left(-\frac{1}{0.5} + 1 - \frac{3}{5}\right)$$

$$= \frac{4.5}{2}\left(-2 + 1 - \frac{5}{9}\right) + \frac{0.35}{2}\left(-2 + 1 - \frac{3}{5}\right)$$

$$= \frac{4.5}{2}\left(-1 - \frac{5}{9}\right) + \frac{0.35}{2}\left(-1 - \frac{3}{5}\right)$$

$$= \left[-\frac{4.5}{2}\left(1 + \frac{5}{9}\right)\right] + \left[-\frac{0.35}{2}\left(1 + \frac{3}{5}\right)\right]$$

$$= \left[-\frac{1}{2}(1 + \text{D.F.}) \text{ x C / O}\right] + \left[-\frac{1}{2}(1 + \text{D.F.}) \text{ x C/O}\right]$$

As $\frac{5}{9}$ and $\frac{3}{5}$ are the distribution factors at the right hand support and left hand support respectively; 4.5 and 0.35 are the C/O at the right hand support and left hand support respectively. Therefore, they are the adjustment or correction due to right hand support and the adjustment or correction due to the left hand support. These adjustments or corrections added to midspan moment to obtain maximum midspan moment.

THE METHOD OF CALCULATING MAXIMUM MIDSPAN MOMENT IS AS FOLLOWS:

Step 1 Write down mid-span moment due to total load as if beam is fully fixed at each end.

$M_{T.L.} = \frac{WL^2}{24}$ for a uniformly distributed load, W.

Step 2 Calculate & write down 'adjustment' due to left hand support.
i.e. Adj. M_l = $-\frac{1}{2}$ (1 + D.F) x C/O

Step 3 Calculate & write down 'adjustment' due to right hand support.
i.e. Adj. M_r = $-\frac{1}{2}$ (1 + D.F) x C/O

Step 4 Add both adjustments to midspan moment to obtain max. midspan moment.

MINIMUM MOMENT

Span BC is shorter than the adjacent spans. It is possible that negative moments may extend across the shortest span. It is therefore necessary to calculate the minimum moment at midspan of span BC.

The loading in Figure 8(a) has dead load only on span BC and total load on the adjacent spans. The procedure is the same as that described in previous sections. The results show that a minimum moment of -7.06 at midspan. This shows that the entire span BC is hogging throughout and reinforcement must be provided accordingly for concrete beam.

	A – B	B – C	C – D	D – E
LOAD PATTERN	T.L.	D.L.	T.L.	D.L.
D.F.	1 2/5	3/5 5/9	4/9 6/11	5/11 0
$M_{D.L}$ or $M_{T.L.}$	-67.5 -67.5	-20 -20	-46.88 -46.88	-45 -45
C/O		+7.47(y) +14.25	(z)	

Midspan M		+10		
Adj. M_l		-5.98 (a1)		
Adj. M_r		-11.08 (b1)		
Min. Midspan M		-7.06		

Figure 8(a)

(y) C/O = (-20 + 46.88) x 5/9 x $^1/_2$ = +7.47
(z) C/O = (-20 + 67.5) x 3/5 x $^1/_2$ = +14.25
(a1) Adj. M_l = - $^1/_2$ (1+3/5) (7.47) = -5.98
(b1) Adj. M_r = - $^1/_2$ (1+5/9) (14.25) = -11.08

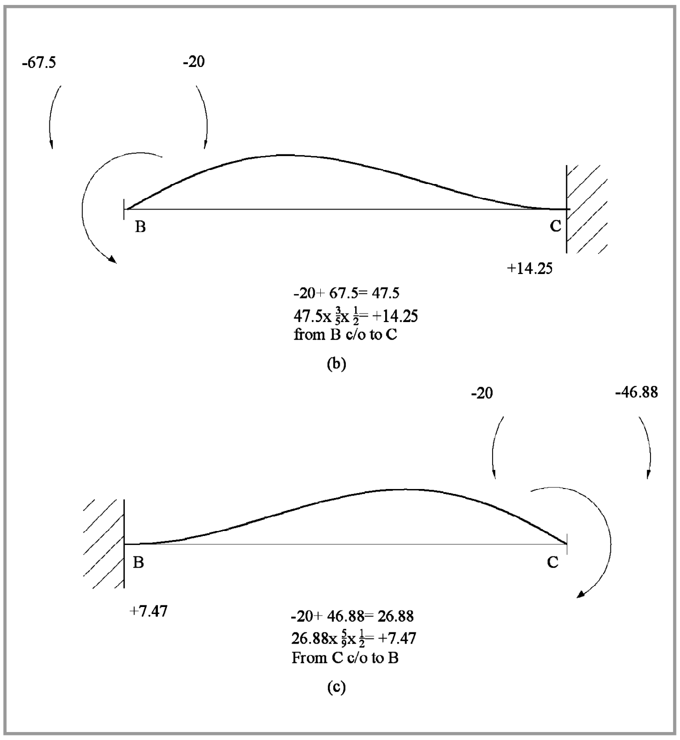


Figure 8: Minimum moment at midspan

Hence, minimum moment at midspan is obtained by adding the adjustments due to right hand support and due to left hand support to midspan moment.

Figure 9 is a sub-frame of a multi-storey structure. The calculations and results of the maximum moments at supports and spans in beams of the sub-frame is as shown in Fig. 10 as against the result for a continuous beam over supports providing no restraint to rotation in Figure 5. A check on span BC shows that the minimum moment at midspan is +2.85.

DETERMINATION OF COLUMN MOMENTS

For multi-storey buildings, it is considered satisfactory to compute column moments under the same assumption used for beam moments i.e. far ends of columns are fixed above and below the floor at which moments are to be determined. Column moments are computed for unbalanced floor loading, that is live load on one side only.

In Figure 9, live load is placed on the alternate spans as shown on load pattern A and load pattern B.

THE METHOD OF CALCULATING THE MAXIMUM COLUMN MOMENTS :

Step 1 Write down the distribution factors for the columns and for the beams and the $M_{D.L}$ and $M_{T.L}$ according to the load pattern.

Step 2 Calculate and write down the carry-over moment (C/O) and C/O = $^1/_2$ x D.F x ($M_{T.L}$ - $M_{D.L}$)

Step 3 Obtain total unbalanced moment (∑M) by adding $M_{T.L}$ to C/O

Step 4 Maximum column moments are obtained by multiplying the difference of the beam moments at the joint by the distribution factors of the columns.

The results of the column moments are shown in Figure 9(a). A summary of the maximum moments obtained using the 2- Cycle Moment Distribution is given as in Figure 9(b) whereas the maximum column moments determined by computer analysis is as shown in Figure 9(c). It can be seen that the differences between the maximum column moments obtained by the two methods are small in values. It is important to note that the deflections of columns follow the direction of the beam unbalanced moment at the joint but the column moments rotate in the opposite direction and bending moment diagram should be drawn on the tension face of a member in normal practice.

It can be seen that the maximum column moments in columns A, B and C are obtained from load pattern A, whereas load pattern B gives maximum column moments in column D.

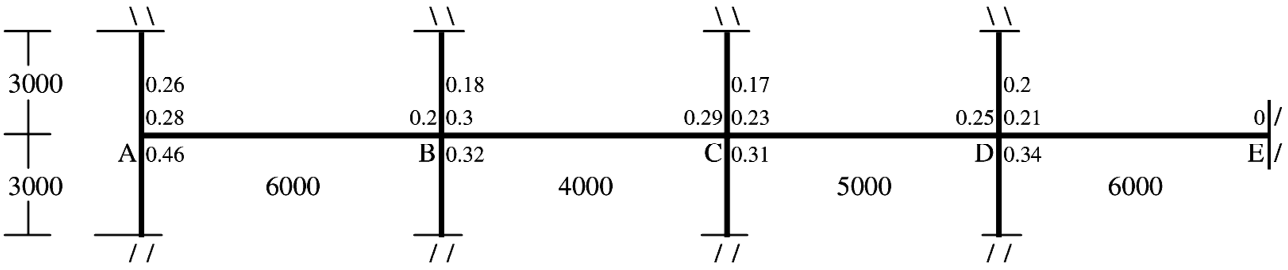
ACCURACY OF THE 2-CYCLE METHOD AGAINST COMPUTER PROGRAMME

For a continuous beam over supports providing no restraint to rotation as shown in Figure 5 the difference between the 2-Cycle Method and a computer programme is 11% maximum for maximum support moments, whereas for maximum midspan moments the difference is 4% maximum for end spans , and is 17% maximum for the internal spans. However, the author's experience confirms that the difference gets smaller when the live load is increased.

The results for maximum moments (support and midspan) in beams of frame in Figure 10 show that the difference between the 2-Cycle Method and a computer programme is negligible.

For maximum column moments in Figure 9, the difference between the 2-Cycle Moment Distribution and a computer programme is 12% maximum, whereas, the difference is as large as 150% in column C under load pattern B. However, it can be ignored as the moment is very small.

In addition to continuous beams and multi-storey building frames subject to vertical loads, the 2-Cycle Method can be used to analyse other types of structures such as continuous beams subject to settlement at support, portal frames and other rigid frames subject to horizontal and vertical loads. And even frames with side sway can be solved by this method. After going through the method and its application properly, one will realise

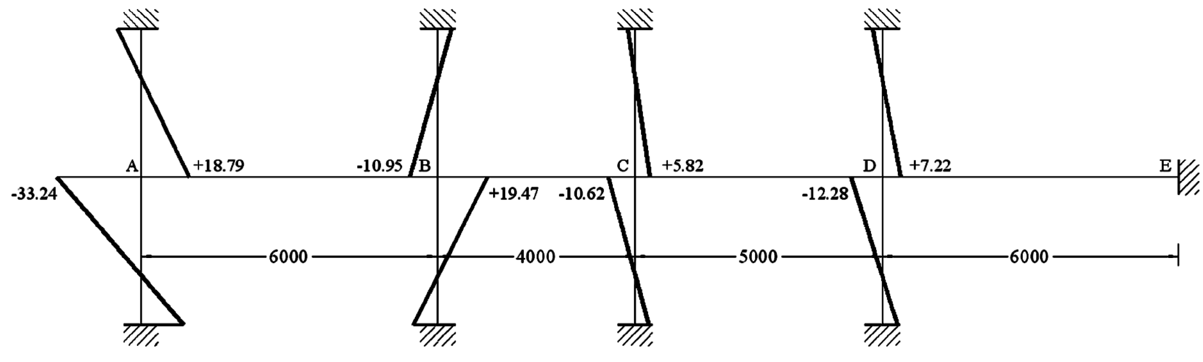


D.L.= 1.0G = 15kN/m,   T.L.= 1.4G+ 1.6Q= 22.5kN/m
Beam- 600x230, Col up- 460x230, Col low- 560x230

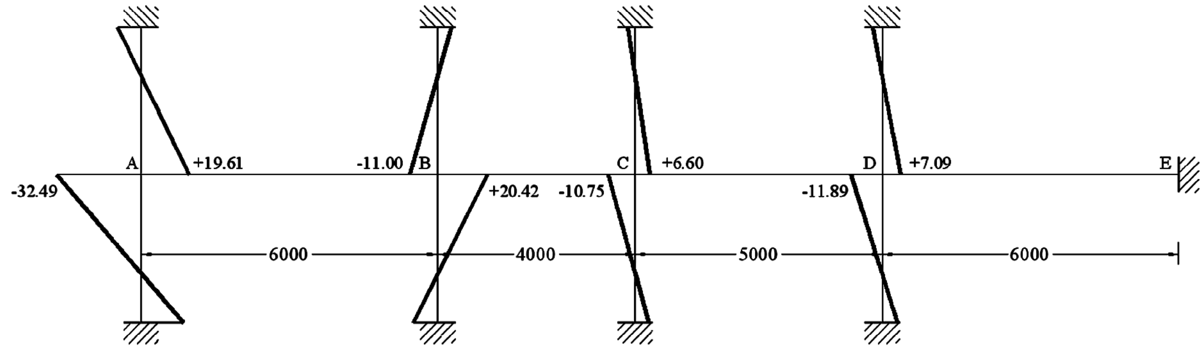
Load Pattern A		T.L.		D.L.		T.L.		D.L.	
D.F.	Upper Col	0.26	0.18		0.17		0.2		
	Lower Col	0.46	0.32		0.31		0.34		
	Beam	0.28	0.2	0.3	0.29	0.23	0.25	0.21	0
$M_{D.L.}$				-20	-20			-45	-45
$M_{T.L.}$		-67.5	-67.5			-46.88	-46.88		
C/O		-4.75	-9.45	+3.9	+7.13	-0.24	-0.39	0	-0.2
ΣM		-72.25	-76.95	-16.1	-12.87	-47.12	-49.97	-45	-45.2
M	Upper Col	+18.79	-10.95			+5.82	-0.99		
		(72.25x 0.26)	(76.95- 16.10)x 0.18			(47.12- 12.87)x 0.17	(49.97-45)x 0.2		
	Lower Col	-33.24	+19.47			-10.62	+1.69		
		(-72.25x 0.46)	(76.95- 16.10)x 0.32			(47.12- 12.87)x 0.31	(49.97- 45)x 0.34		
Computer	Upper Col	+19.61	-11.00			+6.60	-1.00		
	Lower Col	-32.49	+20.42			-10.75	+2.21		
Ratio		0.96	1.00			0.88	0.99		
		1.02	0.95			0.99	0.76		

Load Pattern B		D.L.		T.L.		D.L.		T.L.	
D.F.	Upper Col	0.26	0.18		0.17		0.2		
	Lower Col	0.46	0.32		0.31		0.34		
	Beam	0.28	0.2	0.3	0.29	0.23	0.25	0.21	0
$M_{D.L.}$		-45	-45			-31.25	-31.25		
$M_{T.L.}$				-30	-30			-67.5	-67.5
C/O		-1.5	-6.3	+0.18	+2.25	+4.53	-0.14	0	-3.81
ΣM		-46.5	-51.3	-29.82	-27.75	-31	-31.39	-67.5	
M	Upper Col	+12.09	-3.86		+0.55		7.22		
		(46.5x 0.26)	(51.3- 29.82)x 0.18		(31- 27.75)x 0.17		(67.5- 31.39)x 0.2		
	Lower Col	-21.39	+6.87		-1.01		-12.28		
		(46.5x 0.46)	(51.3- 29.82)x 0.32		(31- 27.75)x 0.31		(67.5- 31.39)x 0.34		
Computer	Upper Col	+12.63	-3.59		+0.22		+7.09		
	Lower Col	-20.64	+7.32		+0.42		-11.89		
Ratio		0.96	1.08		2.5		1.02		
		1.04	0.94		2.4		1.03		

Figure 9(a): Column moments by the 2-cycle moment distribution



(b) Maximum column moments determined by the 2- Cycle Moment Distribution



(c) Maximum column moments determined by computer analysis

Figure 9(b,c): Maximum column moments

Beam size - 600 (h) x 230 (b)

	6000		4000		5000		6000	
	A	B	B	C	C	D	D	E
D.F.	0.28	0.2	0.3	0.29	0.23	0.25	0.21	0
$M_{D.L.}$	-45	-45	-20	-20	-31.25	-31.25	-45	-45
$M_{T.L.}$	-67.5	-67.5	-30	-30	-46.88	-46.88	-67.5	-67.5
C/O	(a) -4.75	(b) -9.45	(c) +0.18	(d) +2.25	(e) -0.25	(f) -3.09	(g) 0	(h) -3.81
ΣM	-72.25	-76.95	-29.82	-27.75	-47.13	-49.97	-67.5	-71.31
Bal. M	(i) +20.23	(j) +9.43	(k) -14.14	(l) -5.62	(m) +4.46	(n) -4.38	(o) +3.68	(p) 0.00
Max Supp M	-52.02	-67.52	-43.96	-33.37	-42.67	-54.35	-63.82	-71.31
Computer	-52.80	-68.10	-43.80	-32.30	-42.50	-55.00	-63.90	-71.30
Ratio	0.99	0.99	1.00	1.03	1.00	0.99	1.00	1.00
Midspan $M_{T.L.}$	+33.75		+15.0		+23.44		+33.75	
Adj. M_l	+3.04 (q)		-0.12 (s)		+0.15 (u)		0.00 (w)	
Adj. M_r	+5.67 (r)		-1.45 (t)		+1.93 (v)		+1.90 (x)	
Max Midspan M	+42.46		+13.43		+25.52		+35.65	
Computer	+42.10		+13.40		+25.90		+35.60	
Ratio	1.01		1.00		0.99		1.00	

Figure 10: Max. M in beams of frame shown in Figure 9

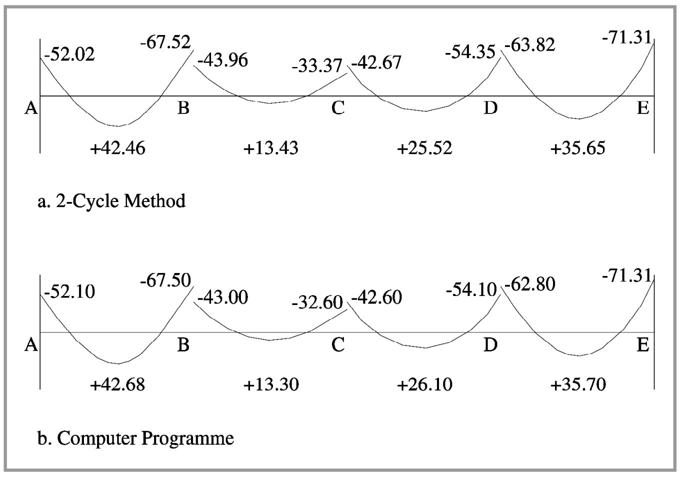


Figure 11: Moment envelop diagram in beams of frame in Figure 9

C/O =	1/2 x D.F.	x $(M_{T.L.}$ - $M_{D.L.})$	
(a) =	1/2 x 0.2	x (-67.5 + 20)	= -4.75
(b) =	1/2 x 0.28	x (-67.5 + 0)	= -9.45
(c) =	1/2 x 0.29	x (-30 + 31.25)	= +0.18
(d) =	1/2 x 0.3	x (-30 + 45)	= +2.25
(e) =	1/2 x 0.25	x (-46.88 + 45)	= -0.25
(f) =	1/2 x 0.23	x (-46.88 + 20)	= -3.09
(g) =	1/2 x 0	x (-67.5 + 0)	= 0
(h) =	1/2 x 0.21	x (-67.5 + 31.25)	= -3.81

Balacing Moments are of opposite sign to reduce larger moment:

(i) =	-(-72.25)	x 0.28	= 20.23
(j) =	-(-76.95 + 29.82)	x 0.20	= +9.43
(k) =	+(-76.95 + 29.82)	x 0.30	= -14.14
(l) =	-(-27.75 + 47.12)	x 0.29	= -5.62
(m) =	+(-27.75 + 47.12)	x 0.23	= +4.46
(n) =	-(-49.97 + 67.5)	x 0.25	= -4.38
(o) =	+(-49.97 + 67.5)	x 0.21	= +3.68
(p) =	+(-71.31 - 0)	x 0	= 0

Adjustments of Span Moment due to Supports Moments:

- 1/2 x (1.0 + D.F.) x C/O

(q) =	- 1/2 x (1 + 0.28)	x -4.75	= +3.04
(r) =	- 1/2 x (1 + 0.2)	x -9.45	= +5.67
(s) =	- 1/2 x (1 +0.3)	x +0.18	= -0.12
(t) =	- 1/2 x (1+0.29)	x +2.25	= -1.45
(u) =	- 1/2 x (1 + 0.23)	x -0.24	= +0.15
(v) =	- 1/2 x (1 + 0.25)	x -3.09	= +1.93
(w) =	- 1/2 x (1 + 0.21)	x 0	= 0
(x) =	-1/2 x (1 + 0)	x -3.81	= +1.91

how useful it is. Its importance just cannot be over-emphasised. Structural engineering is an art of science; it is more an art than a science. Hence, structural engineers are strongly recommended to learn and use this useful technique. In order to have a good understanding of the method, readers are advised to read this article thoroughly.

CONCLUSIONS

Moment coefficients may be used only if loads and spans meet the code requirement. The Cross Method is too time-consuming. It can give support moment only on one load case at a time and the process of moment distribution is repeated until the moments to be carried over are small enough to be neglected. Whereas the 2-Cycle Method is comparatively simpler and faster; not only support moment but also span moment and column moment can be obtained fairly quickly by using this method. A structure basically consists of beams and columns, through the 2- Cycle Method, one can feel, understand and appreciate structures better and thus gain confidence in structural design. The 2-Cycle Method can be used to counter check computer software as no structural engineer should ever use unfamiliar software without applying some verification. Hence, it helps an ~~engineer to be a computer- aided designer rather than just~~ a computer operator. ■

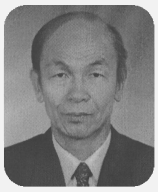
REFERENCES

[1] Portland Cement Association, Continuity in Concrete Building Frames Practical Analysis for Vertical Load and Wind Pressure, 1959.

[2] Chan Kam Leong, “Two Cycle Moment Distribution”.
Bulletin of the Institution of Engineers, Malaysia, August, 1978.

[3] Leong Tuck Wah, “Analysis of Multi-Storey Frame Structures Using The Two Cycle Moment Distribution Method.” Bulletin of the Institution of Engineers, Malaysia, December 1978.

PROFILE



Engr. K. L. Chan obtained his B.Sc (Eng) from the National Cheng Kung University, Taiwan and M.Sc in Construction Management from the University of Bath, England. He is Chartered Civil & Structural Engineer, C.Eng and P.Eng. He is a MICE, MIStruct E, MIEM and MACEM.

He has more than 30 years of experience in C&S works and has worked in Malaysia, Singapore and the U.K. He is the founder of K.L. Chan & Associates, consulting engineers and a practising consulting C&S engineer.