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On Analysis of Invariant Characteristic for Moment Invariant Techniques

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ABSRACT

In this paper a set of equation known as Invariant Error Computation (IEC) is introduced that is used to examine directly the invariant performance properties of moment invariant techniques. The technique consists of a set of equations known as Total Percentage Min Absolute Error (TPMAE), Percentage Min Absolute Error 1 (PMAE1), Percentage Min Absolute Error 2 (PMAE2) and Percentage Absolute Error (PAE). These equations are utilized to measure the similarity between different feature vectors produced by the moment techniques studied. In order to evaluate the effectiveness of the IEC, we examine the invariant properties of three moment techniques; namely; Zernike Moment Invariant (ZMI), Legendre Moment Invariant (LMI), Krawtchouk Moment Invariant (KMI). It is found that Krawtchouk Moment Invariant (KMI) generated the lowest error value for the IEC when compared to ZMI and LMI. For instance PMAE1 for KMI is the lowest with 0.1%-0.5% of error while LMI 8%-25% and ZMI 8%-38% consecutively. Similarly with PMAE2 results, KMI error is between 0.001%-0.9% while for LMI and ZMI is 0.5%-40% and 0.3%-44%. We demonstrate the effectiveness of IEC in examining the invariant properties of the moment techniques.

Keywords: Geometric Moment Invariant (GMI), Zernike Moment Invariant (ZMI), Legendre Moment Invariant (LMI), Krawtchouk Moment Invariant (KMI), Invariant Error Computation (IEC).

INTRODUCTION

The history of moment invariant technique started back almost more than four decades ago in 1962, when M.K. Hu introduced the Geometric Moment Invariant (GMI) which was developed based on the theory of algebraic invariants and moment function. The concept is used to represent the global shape feature of a particular object regardless of position, scaled and rotation factors. In his paper, he had demonstrated how this technique can be applied for alphabet shape representation.

Since then, many researchers had tried to improve its fundamental concept to make it more effective in terms of invariant capabilities and information redundancy [Yap, P. T *et al.* (2003)]. Basically, two main improvements had been made to the moment invariant computation. First is the introduction of more effective kernel function such as the use of orthogonal polynomial as the kernel function and second is the invariant characteristics enhancement [Yap, P.T. *et al.* (2003)]. These improvements spur to the development of new moment invariant techniques such as Zernike Moment Invariant (ZMI), Legendre Moment Invariant (LMI) and Krawtchouk Moment Invariant (KMI). In order to drive the level of knowledge beyond current frontiers we examine closely the invariant properties of ZMI, LMI and KMI techniques using the IEC developed.

Since moment techniques preserve the invariant properties against translation, position and rotation, therefore the feature vector produce for both original images and its counterpart variation should have similarities in their values. Thus, Puteh B. Saad (2004) had introduced an effective technique to measure this similarity also known as Min Absolute Error (MAE). She has demonstrated that ZMI is less sensitive compared to Geometric Moment Invariant (GMI) in binary trademarks images.

Therefore, in order to examine directly of the invariant performance capabilities, we have introduced a set of equation that is known as Invariant Error Computation (IEC). This brand new technique consists of four useful computations: Total Percentage Min Absolute Error (TPMAE), Percentage Min Absolute Error 1 (PMAE1), Percentage Min Absolute Error 2 (PMAE2) and Percentage Absolute Error (PAE). These equations were used to measure the similarity between different feature vectors produced from all the moment techniques studied that represent the same object of the particular image.

The rest of this paper is organized as follows: Section II explains ZMI, LMI and KMI as features extraction method. Section III describes the invariant error computation (IEC) and section IV presents the overall methodology. Section V is on Results and Discussion and the paper ends with a conclusion in Section VI.

MOMENT INVARIANT

Zernike Moment Invariant (ZMI)

Zernike Moments were first introduced by Teague (1980), based on continuous orthogonal functions called Zernike polynomials. Equation (1) provides a convenient way to express Zernike moments in terms of geometric moments in Cartesian form. Then Zernike Moment invariant (ZMI) functions are derived from equation (3) which is invariant against rotation and scaling factors. f(x,y) refer to the pixel density of N x N image size.

$$Z_{mn} = \frac{n+1}{\pi} \sum_{k=m}^{n} B_{nmk} \sum_{x=1}^{N} \sum_{y=1}^{M} \left(x - iy\right)^{m} \left(x^{2} + y^{2}\right)^{(k-m)/2} f\left(x, y\right) \tag{1}$$

$$B_{nmk} = \frac{(-1)^{(n-k)}\, 2\left(\frac{n+k}{2}\right)!}{\left(\frac{n-k}{2}\right)!\left(\frac{k+m}{2}\right)!\left(\frac{k-m}{2}\right)!} \tag{2}$$

$$\varphi_1 = Z_{p0} \quad ; \quad \varphi_2 = \left|Z_{pq}\right|^{2} \tag{3}$$

Legendre Moment Invariant

The Legendre moment was also introduced by Teague (1980) which is produced based on Legendre polynomials. The Legendre moments of order (p+q) can be expressed in terms of geometric moments as shown in equation (4) whereas in equation (5) $|x| \leq 1$ and (n-k) is even. The purpose of v_{pq} is to give TMI equation invariant against translation, scaling
and rotation factors.

$$L_{pq} = \frac{(2p+1)\ (2q+1)}{4} \sum_{i=0}^{p} \sum_{j=0}^{q} a_{pi} a_{qj} m_{ij} \tag{4}$$

$$a_{pi} = P_n\left(x\right) = \sum_{k=0}^{n} \left(-1\right)^{(n-k)/2} \frac{1}{2^{n}} \frac{(n+k)!\, x^{k}}{\left(\frac{n-k}{2}\right)!\left(\frac{n+k}{2}\right)!\, k!} \tag{5}$$

$$v_{pq} = M_{00}^{-\gamma} \sum_{x=1}^{N} \sum_{y=1}^{N} \left[\left(x - \bar{x}\right) \cos\phi + \left(y - \bar{y}\right) \sin\phi\right]^{p} \times \left[\left(y - \bar{y}\right) \cos\phi - \left(x - \bar{x}\right) \sin\phi\right]^{q} f\left(x, y\right) \tag{6}$$

Where: $$\gamma = \frac{n+m}{2} + 1 \qquad (7)$$

$$\phi = 0.5 \tan^{-1} \frac{2\mu_{11}}{\mu_{20} - \mu_{02}} \qquad (8)$$

Krawtchouk Moment Invariant (KMI)

Krawtchouk moment invariants were derived by P.T *Yap et al.* (2003) using the concept of Krawtchouk polynomial function with the implementations of linear combinations of Geometric Moment. The (p+q) order of Krawtchouk moment is given by (9).

$$\tilde{Q}_{nm} = \Omega_{nm} \sum_{i=0}^{n} \sum_{j=0}^{m} a_{i,n,p1} a_{j,m,p2} \tilde{v}_{ij} \qquad (9)$$

$$\Omega_{nm} = \left[\rho(n; p_1, N-1) \rho(n; p_2, N-1) \right]^{-0.5} \qquad (10)$$

$$\rho(n; p, N) = (-1)^n \left(\frac{1-p}{p} \right)^n \frac{n!}{(-N)_n} \qquad (11)$$

$$\sum_{k=0}^{n} a_{k,,n,p} x^k = \sum_{k=0}^{n} \frac{(-n)_k (-x)_k}{(-N)_k} \times \frac{p^{-k}}{k!} \qquad (12)$$

$$\tilde{v}_{ij} = \sum_{p=0}^{i} \sum_{q=0}^{j} \binom{i}{p} \binom{j}{q} \left(\frac{N^2}{2} \right)^{\frac{p+q}{2}+1} \left(\frac{N}{2} \right)^{i+j-p-q} v_{pq} \qquad (13)$$

$$\binom{x}{y} = \frac{x!}{(x-y)!x!} \qquad (14)$$

Invariant Error Computation (IEC)

In order to analyze between the original object with their counterparts variations, a series of equations is introduce to measure the similarities between them, that is also known as IEC. First of all, given the features vector for original image, $H_a(\gamma_i) = \{\gamma_i, \gamma_{i+1}, \gamma_{i+2} \ldots \gamma_n\}$ where a is referred as the class name with i as the feature dimension. Each class consists of a set of images produced by transforming the original image with different scale and orientations. Thus, each features vector for the variations of images can be presented using $F_m^a(\gamma_i) = \{\gamma_i, \gamma_{i+1}, \gamma_{i+2} \ldots \gamma_n\}$ where m refers to the type of variations of class a. As a result, for one object we have several feature vectors representing the images with different scale and orientations.

The absolute error for each dimension is calculated using the equation (15). Using (16) then the Percentage of Absolute Error (PAE) of each pattern of class a is computed. Furthermore, using the previous equation, the Percentage of Mean Absolute Error (PMAE1) for the feature vector for the variations of class a can be calculated as shown in (17) where it is adopted in order to examine the error distribution with different types of perturbation. M refers to the number of variation of class a. While PMAE2 is used to examine an error distribution along the dimension of each features vectors as described in equation (18). Finally, the total error of class a for each method can be presented by the total Percentage of Mean Absolute Error (TPMAE) using equation (19) and (20).

$$AE_m^a(\gamma_i) = \Delta_m^a \gamma_i = \left| H_a(\gamma_i) - F_m^a(\gamma_i) \right| \quad (15)$$

$$PAE_m^a(\gamma_i) = \%\Delta_m^a \gamma_i = \frac{\Delta_m^a \gamma_i}{\left| H_a(\gamma_i) \right|} \times 100 \quad (16)$$

$$PMAE1_m^a(\gamma) = \frac{1}{I} \sum_{i=1}^{I} PAE_m^a(\gamma_i) \quad (17)$$

$$PMAE2^a(\gamma_i) = \frac{1}{M} \sum_{m=1}^{M} PAE_m^a(\gamma_i) \quad (18)$$

$$TPMAE_a = \frac{1}{M} \sum_{m=1}^{M} PMAE1_m^a(\gamma) \quad (19)$$

$$TPMAE_a = \frac{1}{n} \sum_{i=1}^{n} PMAE2^a(\gamma_i) \quad (20)$$

where: $$\frac{1}{M}\sum_{m=1}^{M} PMAE1_m^a(\gamma) = \frac{1}{n}\sum_{i=1}^{n} PMAE\,2^a(\gamma_i)$$

METHODOLOGY

In this work we have used 240 binary images that represent 20 different insects. Figure 1 illustrates an example of binary images that were used, while their scaling and rotation factors are shown in Table 1.

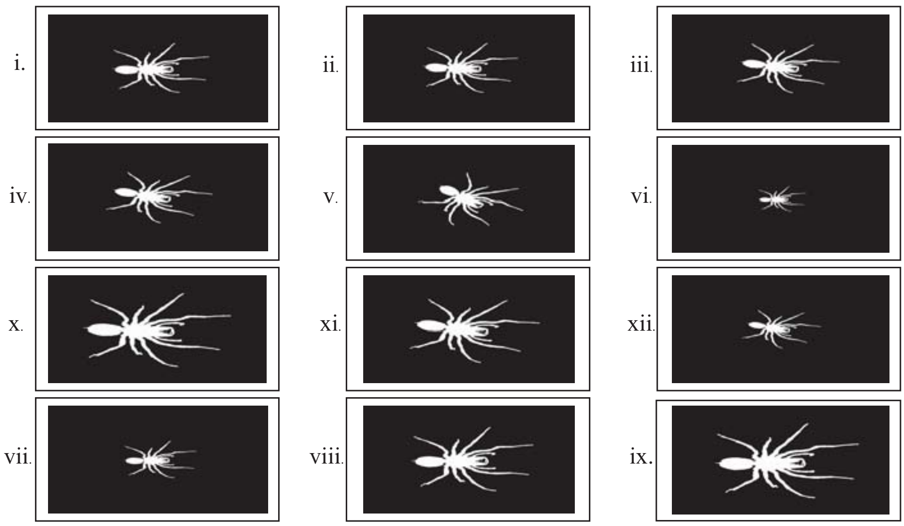

Figure 1. An example of images been used with its various rotation and scaling factor.

Then feature vectors are extracted from these images using three types of moment techniques. Consequently, feature vectors are analyzed to examine their invariant characteristics. The rest of insect images are shown in Figure 7.

RESULT AND DISCUSSION

Original Features

As we can see from all the tables, different moment method will generate a dissimilar value of feature vectors for the same image. Therefore, each group of data produced by different techniques definitely has its own feature space. Note that, KMI generated a huge number while others are not. This is due to the fact of using the equation (13) which been applied in order to bring the invariant properties into the Krawtchouk Moment [Yap,P.T. *et al.* 2003]

Table 1 ZMI feature vector.

Features	Z20	Z22	Z31	Z33	Z42	Z40
Original	-0.42318	0.03541	0.00607	0.00062	0.07360	0.18108
5	-0.43284	0.03370	0.00463	0.00048	0.07679	0.18100
15	-0.43290	0.03371	0.00460	0.00055	0.07678	0.18114
20	-0.43297	0.03369	0.00465	0.00064	0.07683	0.18115
45	-0.43426	0.03349	0.00457	0.00134	0.07732	0.18102
0.5X	-0.44494	0.03175	0.00321	0.00029	0.08001	0.18378
0.75X	-0.43740	0.03296	0.00407	0.00039	0.07812	0.18177
13X	-0.43308	0.03366	0.00465	0.00045	0.07683	0.18098
15X	-0.43329	0.03367	0.00453	0.00045	0.07690	0.18142
5+15X	-0.43617	0.03316	0.00421	0.00042	0.07781	0.18137
15+13X	-0.43668	0.03307	0.00414	0.00049	0.07790	0.18163
20+0.75X	-0.44357	0.03192	0.00339	0.00046	0.07995	0.18250

Table 2 LMI feature vector.

Features	L20	L02	L21	L12	L30	L03
Original	0.04069	0.23775	-0.00341	-0.00104	0.00081	0.04309
5	0.04057	0.23281	-0.00270	-0.00078	0.00084	0.03743
15	0.04054	0.23281	-0.00270	-0.00102	0.00085	0.03732
20	0.04055	0.23276	-0.00270	-0.00101	0.00086	0.03750
45	0.04050	0.23214	-0.00275	-0.00109	0.00091	0.03723
0.5X	0.04023	0.22682	-0.00177	-0.00109	0.00083	0.03084
0.75X	0.04044	0.23056	-0.00239	-0.00101	0.00083	0.03502
13X	0.04058	0.23268	-0.00266	-0.00072	0.00081	0.03749
15X	0.04050	0.23264	-0.00268	-0.00097	0.00084	0.03705
5+15X	0.04048	0.23116	-0.00246	-0.00097	0.00084	0.03561
15+13X	0.04048	0.23089	-0.00233	-0.00069	0.00086	0.03525
20+0.75X	0.04034	-0.22743	-0.0026	-0.00117	0.00089	0.03187

Table 3 Types of variations in Figure 3.

i	: original image
ii	: the image is rotated to 5°
iii	: the image is rotated to 15°
iv	: the image is rotated to 20°
v	: the image is rotated to 45°
vi	: the image is reduced to half of its original size
vii	: the image is reduced to 0.7X of its original size
viii	: the image is enlarged 1.3 X its original size
ix	: the image is enlarged 1.5 X its original size
x	: the image is rotated to 5° and enlarged 1.5 X
Xi	: the image is rotated to 15° and enlarged 1.3 X
Xii	: the image is rotated to 20° and enlarged 0.75 X

Table 4 KMI features vectors.

$x10^5$	K20	K02	K21	K12	K30	K03
Original	1956	2670	213011	253394	-59231	-59234
5	1956	2652	212986	252382	-59231	-59234
15	1955	2652	212981	252381	-59231	-59234
20	1955	2652	212982	252372	-59231	-59234
45	1955	2650	212972	252244	-59231	-59234
0.5X	1954	2630	212917	251154	-59231	-59234
0.75X	1955	2644	212959	251920	-59231	-59234
13X	1956	2651	212987	252355	-59231	-59234
15X	1955	2651	212972	252347	-59231	-59234
5+15X	1955	2646	212967	252044	-59231	-59234
15+13X	1955	2645	212967	251989	-59231	-59234
20+0.75X	19550	2632	212938	251279	-59231	-59234

Table 5 AE for image 17 with 5° and different types of moment.

Moment techniques	1	2	3	4	5	6
ZMI	0.0096	0.00171	0.00144	0.000153	0.0031	0.00008
LMI	0.0001	0.00493	0.00002	0.005658	0.0007	0.0002
KMI	43670	1789013	2464337	10119968	212	41892

Table 6 PAE for image 17 with 5° and different types of moment.

Moment techniques	1	2	3	4	5	6
ZMI	2.23244	1.04564	4.14920	5.07514	21.2193	22.7891
LMI	0.29710	2.12118	3.40154	3.11344	26.1252	20.6291
KMI	0.00036	0.00071	0.01157	0.02233	0.40098	0.67447

Percentage Absolute Error (PAE)

Generally, one way to examine the invariant characteristic of feature vector produced is by calculating the AE. This is because AE describes the difference between an original data with its counterpart's variations. This method is successfully deployed when a comparison is made between variations of data generated by the techniques only. Hence, for this work it is essential to search other flexible method that can be implemented for comparison among different moment techniques.

The reason AE is unsuitable to compare different moment techniques are because it comprises only absolute number and without bringing out the important information from the original data. In addition, each moment technique uses different equations, thus generate dissimilar feature spaces. Therefore, instead of using equation (15), we used equation (16) which refers to Percentage Absolute Error (PAE).

This fact can be clearly seen by understanding the example as follow. Given two original data x_1 = 0.7 and y_1 = 0.002. Then both values are increased to x_2 = 0.8 and y_2 = 0.001. If we implement AE using equation (15), thus the value of AE for x is 0.1 while y equal to 0.001. At this stage, we notice that, AE for x is greater than y. Hence, we can conclude y is more invariant than x. However, when we perform the equation (16), the value of PAE for both x and y is 12.5% and 50% respectively. Different from AE, PAE define that x is more invariant than y. Nevertheless, PAE describe the percentage of changes occurred in the data examine. It can be applied whether in huge or small numbers. AE is not effective to describe the invariant properties especially when the difference between data is large.

Table 5 and 6 illustrate the AE and PAE for image 1 with 5° perturbation. Numbers one (1) to six (6) appear on the top column referred to the feature dimension. The advantage of PAE against the AE is shawn in both tables. For example, PAE for ZMI for the fourth dimension is smaller than LMI of the same dimension. And this fact is in opposite behavior when PAE is adopted in Table 6. Moreover, PAE itself will not bring a lot of information needed. Thus, a combination of set of PAE will contribute another valuable conclusion.

Percentage Min Absolute Error 1 (PMAE1)

The main objective of PMAE1 calculation is to determine the distribution of error occurred among image variation for one object. For example, Figure 2, illustrates the value of PMAE1 versus image variation for the first insect. As we can see, the scaling factor of 0.5X generates the highest error compared to other factors for all moment invariant applied. This circumstance is also known as spatial quantization error [Umbaugh, S.E., 1998].

Figure 3, demonstrates the example which will help to explain clearly the cause of this error. Basically, the black areas represent the object and a white area is the background. The original object as in Figure 3(a) while 3(b) illustrates the image under the influence scaling factor of 0.5X. The error happens when one pixel from the original object (as shown by A) is not situated in Figure 3(b). This example is defined only for simple shape information. Therefore, for complex shape like insect images, this dilemma increases especially when the scaling factor increases.

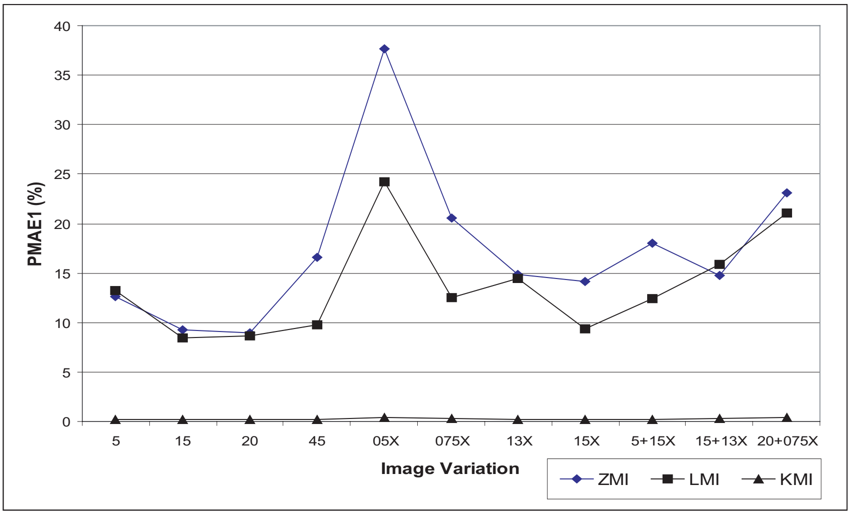

Figure 2. PMAE1 versus image variation for image 17.

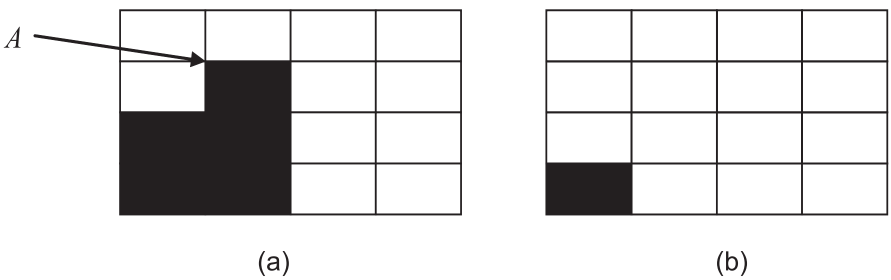

Figure 3. Spatial quantization error.

Nevertheless, ZMI generate the highest PMAE1 for every variation of image one (1). However, KMI produced the lowest error compare to ZMI and LMI. We also found that, when an image is under the influence of both scaling and rotation factor, the value of PMAE1also increases. This can be proven by looking the PMAE1 value for 5+15X is greater than both 5 and 15X for all types of moment invariant.

Percentage Min Absolute Error 2 (PMAE2)

The benefit of calculating PMAE2 using equation (12) is that we can examine an error distribution along the dimension of feature vector. Principally, PMAE2 is a minimum value for every PAE within the same dimension of existing variation. Nevertheless, we had found that, when the order of moment is increased therefore, the value of PMAE2 also become higher. This is the key reason why we just choose until order (3+1) for moment techniques applied.

Mukundan, R. *et al* (1998) in his book highlights that, moment of order higher than four are not commonly used. This is because higher order moments are more sensitive to image noise and quantization effects. This fact is proven by the value of error rising as the dimension increases as shown in Figure 4. At this stage another main factor call discreatization error causes this problem. Liao, S.X. *et al.* (1996) described in details about this dilemma which occurred inside the moment function. Nevertheless, ZMI also demonstrates the highest error compared to other types of moment. While both KMI generate lower value of PMAE2 compare to other four moments.

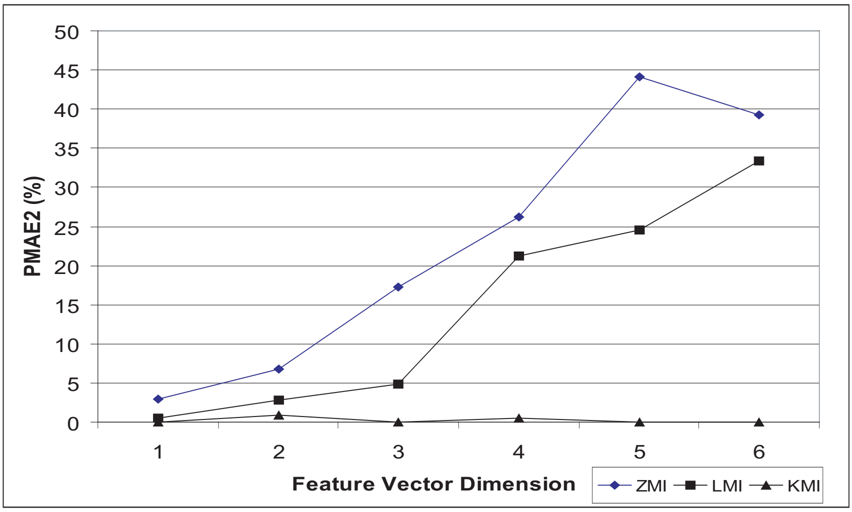

Figure 4. PMAE2 Versus Image Variation For Image 17.

Total Percentage Min Absolute Error (TPMAE)

Finally, the main objective of all types of equation outlined in previous sections is to identify the error for each image by means of different types of moment. This is done by performing equation (20). Figure 5, illustrates TPMAE for every insect images applied in this work.

Furthermore, we found that some of the images generate huge amount of error and others are not. For example, image 5 demonstrates the highest TPMAE value compare with other insect images. Whereas, image 18 shows the opposite side for all moment technique employed. Basically, most of TPMAE value is contributed by the error generated by image with scaling factor of 0.5X. Figures 6 and 7 describe both images and their variation with scaling factor of 0.5X.

From both insect images, we notice that the shape information of image 18 is less complicated than image 5. Therefore, the error for its counterpart variation in Figure 7b is not as huge as in Figure 6b. In other words, image 18 with the scaling factor of 0.5X is losing less the valuable shape information compared to image 5. Again spatial quantization error is the main element causing this dilemma as described in Section 5.3.

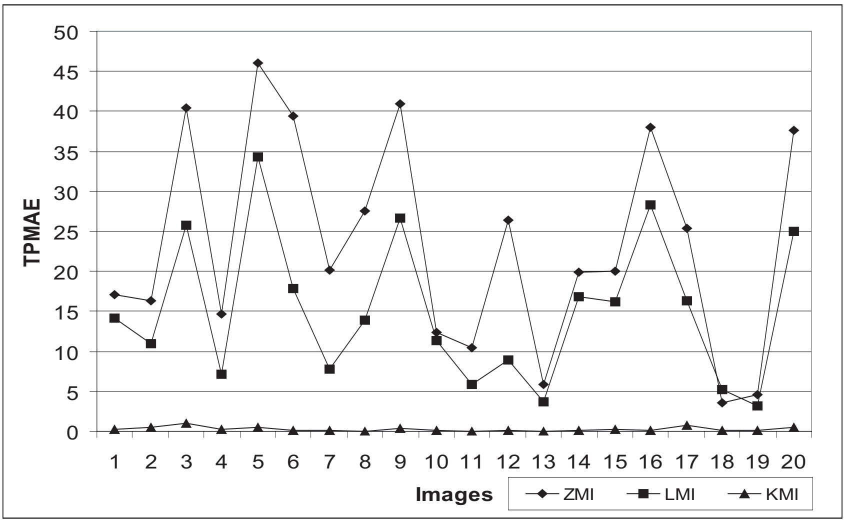

Figure 5. TPMAE versus images.

Figure 6. Image 5 and its variation 0.5X.

Figure 7. Image 18 and its variation 0.5X.

CONCLUSION

In this paper, we have introduced IEC, a set of useful equation consisting of TPMAE, PMAE1, PMAE2 and PAE. These equations were used to measure the similarity between different feature vectors that represent the same object. PMAE1 is used to examine the distribution of error in the feature vectors produced by moment invariant technique compared to various images that represent the same object. In addition, PMAE2 is designed to examine an error distribution along the dimension of feature vectors. However, from both equations we found that, the KMI technique generated feature vectors that are more invariant capability compared to ZMI and LMI since it's produced the lowest value for both PMAE1 and PMAE2. From our observation, it is known that, the value of this error will increase as the number of scaling factors decrease. This circumstance is caused by spatial quantization error. Moreover, we have proved that the value of this error also increase when the number of moment order increase which is known as a discreatization error. Nevertheless, we have demonstrated that, effectiveness of moment in preserving the invariant shape characteristics can be examined using the Invariant Error Computation technique (IEC) directly instead relying with other methods such as neural networks.

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