Effects of Radiation, Joule Heating and Viscous Dissipation on MHD Marangoni Convection over a Flat Surface with Suction and Injection

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Abstract: In this paper, we studied the effects of thermal radiation, Joule heating and viscous dissipation on magnetohydrodynamics (MHD) Marangoni convection boundary layer over a flat surface. We also investigated the influence of suction and injection on the boundary layer. Numerical results were obtained using the shooting method along with the Runge-Kutta-Fehlberg method. The influences of the interest parameters on the reduced velocity along the interface, velocity profiles as well as the reduced heat transfer at the interface and temperature profiles were presented in tables and figures. From the results, we discovered that thermal radiation, magnetic parameter, Joule heating, viscous dissipation and suction parameter can reduce the velocity and heat transfer at the interface.

Key words: Convection・Boundary layer・Flat surface

INTRODUCTION

The magneto-hydrodynamics (MHD) of an electrically conducting fluid has important bearings in geophysics, astrophysics, aeronautics, engineering applications and many other areas. (Khaleque and Samad, [1]). Because of this reason, many researchers tend to apply the MHD flow into their problems. For instance, Amkadni and Azzouzi [2] studied the similarity solution of MHD boundary layer flow over a moving vertical cylinder. Meanwhile, Rajeswari *et al*. [3] analyzed the influence of chemical reaction parameter, magnetic parameter, buoyancy parameter and suction parameter on nonlinear MHD boundary layer flow through a vertical porous surface. Recently, Bhattacharyya and Layek [4] investigated the effects of chemical reaction in MHD boundary layer flow over a permeable stretching sheet subject to suction or injection. They discovered that the rate of solute transfer can be enhanced by increasing the values of the magnetic parameter and suction parameter. Moreover, Al-Mudhaf and Chamkha [5] investigated the MHD thermosolutal Marangoni convection boundary layer. They have studied the effects of heat generation/absorption, Hartmann number, the chemical reaction parameter and the suction/injection parameter on the flow.

The study of thermal radiation has received considerable attention in recent years as its effect in high operating temperature process. Pathak and Maheshwari [6] have analyzed the influence of radiation on an unsteady free convection flow bounded by an oscillating plate with variable wall temperature. The effects of thermal radiation, buoyancy and suction/blowing on natural convection heat and mass transfer over a semi-infinite stretching surface has been studied by Shateyi [7]. Moreover, Suneetha *et al*. [8] have investigated the radiation effects on the MHD free convection flow past an impulsively started vertical plate with variable surface temperature and concentration. The effects of Joule heating and viscous dissipation are usually characterized by the Eckert number and magnetic parameter. Both have a very important part in geophysical

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flows and in nuclear engineering. (Alim *et al.*, [9]). Not only that, the effects of suction or injection on boundary layer flow also have a huge influence over the engineering application and have been widely investigated by numerous researchers. With this understanding, many researchers studied the effects of Joule heating and viscous dissipation along with the effects of suction or injection in various geometries; for example, Borisevish [10] for heat transfer near a rotating disk, Duwairi [11] and Chen [12] for MHD convection flow in the presence of radiation, Turkyilmazoglu [13] for viscous incompressible Newtonian and electrically conducting fluid flow over a porous rotating disk.

This paper presents the combined effects of radiation, Joule heating and viscous dissipation on MHD Marangoni convection flow in the presence of suction or injection. The objectives of this paper are to investigate the effects of radiation parameter, magnetic parameter, Eckert number as well as suction or injection parameter on the surface velocity, surface temperature gradient as well as the velocity and temperature profiles. The numerical results are obtained using the shooting method along with the Runge-Kutta-Fehlberg method.

Mathematical Formulation: Consider the steady, two dimensional, laminar boundary layer flow of an electrically-conducting fluid over a flat surface through a uniformly distributed transverse magnetic field of strength B_0. With the usual boundary layer approximations, the governing equations can be written in the following form (Khaleque and Samad, [1]);

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=v\frac{\partial^2 u}{\partial y^2}-\Delta\frac{B_0^2}{\rho}u, \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y}+\frac{\sigma B_0^2}{\rho c_p}u^2+\frac{v}{c_p}\left(\frac{\partial u}{\partial y}\right)^2, \tag{3}$$

subject to the boundary conditions,

$$v(x,0)=-v_w,\ T(x,0)=T_\infty+Ax^2,\ \mu\frac{\partial u}{\partial y}=-\frac{d\sigma}{dT}\frac{\partial T}{\partial x}\ \text{at}\ y=0$$
$$u(x,\infty)=0,\ T(x,\infty)=T_\infty\ \text{ as }\ y\to\infty \tag{4}$$

where u and v are the velocity components in the x and y directions respectively, υ is the kinematic viscosity, Δ is the electric conductivity, B_0 is the uniform magnetic field strength, ρ is the density of the fluid, T is the fluid temperature, k is the thermal conductivity of the fluid, c_p is the specific heat at constant pressure, q_r is the radiative heat flux, v_w is the constant suction ($v_w > 0$) or injection ($v_w < 0$) velocity and μ is the dynamic viscosity. When we use the Rosseland approximation for radiation, the radiative heat flux can be simplied as, (Brewster, [14])

$$q_r=-\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \tag{5}$$

where σ^* is the Stefan-Boltzmann constant and k^* is the mean absorption coefficient. Then we assume that the temperature differences within the flow such the term T^4 may be expressed as a linear function of temperature. Hence, neglecting higher-order terms we obtain

$$T^4\approx 4T_\infty^3T-3T_\infty^4 \tag{6}$$

Using (5) and (6), equation (3) reduces to (Ishak, [15])

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha(1+Nr)\frac{\partial^2 T}{\partial y^2}+\frac{\sigma B_0^2}{\rho c_p}u^2+\frac{v}{c_p}\left(\frac{\partial u}{\partial y}\right)^2, \tag{7}$$

where $\alpha = k/(\rho c_p)$ is the thermal diffusivity and $Nr=16\sigma^*T_\infty^3/(3kk^*)$ is the radiation parameter. Further, we use the similarity transformation by Al-Mudhaf and Chamkha [5] and the standard definition of the stream function such that $u = \partial\Psi/\partial y$ and $v = \partial\Psi/\partial x$ to obtain the similarity solution of the problems. We apply the following similarity transformation

$$f(\eta)=C_2x^{-1}\psi(x,y),\ \theta(\eta)=\frac{[T(x,y)-T_\infty]x^{-2}}{A},\eta=C_1y \tag{8}$$

where

$$C_1=\sqrt[3]{\frac{\rho A(d\sigma/dT)}{\mu^2}},C_2=\sqrt[3]{\frac{\rho^2}{\mu A(d\sigma/dT)}}, \tag{9}$$

into equations (1), (2) and (7) and get the following dimensionless equations;

$$f'''+ff''-(f')^2-M^2f'=0, \tag{10}$$

$$\frac{(1+Nr)}{\Pr}\theta''-2f'\theta+f\theta'+Ec[M^2(f')^2+(f'')^2]=0, \tag{11}$$

with boundary conditions,

$$f(0) = f_w,\ \ f''(0) = -2,\ \ \theta(0) = 1,$$
$$f'(\infty) = 0,\quad \theta(\infty) = 0. \tag{12}$$

where $M^2 = \frac{\Delta B_0^2 C_2}{\rho C_1}$ is the magnetic field parameter, *Pr* is the Prandtl number, $Ec = \frac{C_1^2}{A c_p C_2^2}$ is the Eckert number, $f_w(> 0)$ is the constant suction parameter and $f_w(< 0)$ is the constant injection parameter. It should be noticed that the effects of viscous dissipation is characterized by the Eckert number *Ec* while Joule heating effects is represented by the product of *Ec* and *M*.

RESULTS AND DISCUSSION

The system of equations (10) and (11) with boundary conditions (12) is solved numerically using the shooting method along with the Runge-Kutta-Fehlberg method. Table 1 and 2 show the comparison of $f'(0)$ and $-\theta'(0)$ with those reported by Al-Mudhaf and Chamkha [5], which show an excellent agreement thus give confidence that the numerical results obtained are accurate.

Further, Table 3 shows the effects of Prandtl number *Pr*, magnetic parameter *M*, radiation parameter *Nr*, Joule heating and viscous dissipation on the surface temperature gradient, $-\theta'(0)$ when $f_w = 0$.

It is observed that when Ec = 0 (no Joule heating and viscous dissipation), the increased values of the Prandtl number *Pr* will also increase the surface temperature gradients as well as the interface heat transfer. On the other hand, increasing the values of magnetic parameter *M* and radiation parameter *Nr* tend to reduce the $-\theta'(0)$. When we only consider the viscous dissipation effect, $-\theta'(0)$ increases with the increasing of some particular Prandtl number values but will decrease for higher Prandtl number. Meanwhile reduction of the interface heat transfer can be seen when we increase the values of magnetic parameter *M*, *Ec* and radiation parameter *Nr*. The same trend also occurs when viscous dissipation and Joule heating are included in the problems. Further decrease in $-\theta'(0)$ is observed with the increasing of parameter *M*, *Ec* and *Nr*.

Table 1: The Values of $f'(0)$ and $-\theta'(0)$ for Various Values of *M*.

Al-Mudhaf and Chamkha [5]			Present Shooting Method		Runge-Kutta Method	
M	$f'(0)$	$-\theta'(0)$	$f'(0)$	$-\theta'(0)$	$f'(0)$	$-\theta'(0)$
0	1.587671	1.442203	1.587372	1.442210	1.587372	1.442210
1	1.315181	1.206468	1.314596	1.205839	1.314596	1.206059
2	0.903945	0.759604	0.903212	0.758415	0.903212	0.759065
3	0.644888	0.442240	0.644022	0.445802	0.644022	0.446769
4	0.493358	0.272847	0.492475	0.277440	0.492478	0.287078

Table 2: The Values of $f'(0)$ and $-\theta'(0)$ for Various Values Of f_w

Al-Mudhaf and Chamkha [5]			Present Shooting Method		Runge-Kutta Method	
f_w	$f'(0)$	$-\theta'(0)$	$f'(0)$	$-\theta'(0)$	$f'(0)$	$-\theta'(0)$
-2	2.383451	1.251341	2.382975	1.250618	2.382975	1.250618
-1	2.000379	1.336441	2.000001	1.335852	1.999999	1.335852
0	1.587671	1.442203	1.587400	1.442058	1.587401	1.442067
1	1.179708	1.634990	1.179508	1.634359	1.179509	1.634360
2	0.848026	2.020945	0.847698	2.019459	0.8477075	2.019468

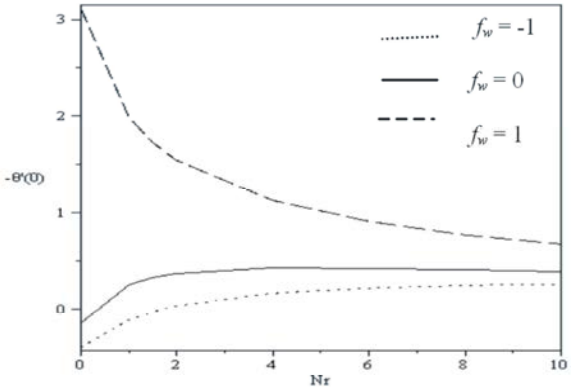

Fig. 1: Variations of radiation parameter, *Nr* with interface heat transfer, $-\theta'(0)$ for different f_w when $Pr = 7$, $M = 0.5$, $Ec = 1$

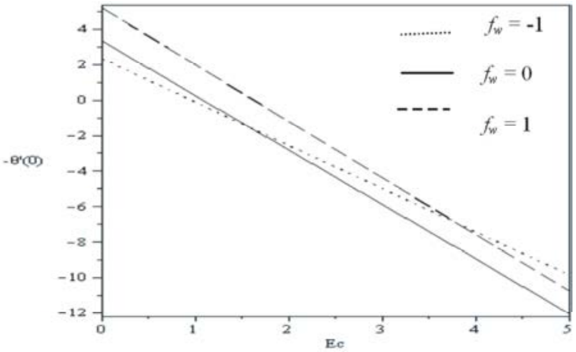

Fig. 2: Variations of Eckert number, *Ec* with interface heat transfer $-\theta'(0)$ for different f_w when $Pr = 7$, $M = 0.5$, $Nr = 1$.

The variations of radiation parameter *Nr* and Eckert number *Ec* with the reduced heat transfer at the interface, $-\theta'(0)$ in the presence of suction and injection parameter are shown in Fig. 1 and Fig. 2, respectively. $f_w = 1$ indicates the suction parameter while $f_w = -1$ refers to injection parameter. Fig. 1 reveals that as *Nr* increase, the interface heat transfer will gradually increase. It is noticed in the figure that the suction parameter increased the interface

Table 3: Effects of *Pr*, *M*, *Nr* and *Ec* on the Heat Transfer at the Interface, $-\theta'(0)$

				$-\theta'(0)$		
Pr	*M*	*Nr*	*Ec*	No Joule and viscous heating (*Ec*=0)	No Joule heating with viscous heating	With Joule heating and viscous heating
0.7 (air)	0.1	1	0	0. 846557		
			1		0.400419	0.397620
			2		-0.045718	-0.051315
		2	0	0.629720		
			1		0.315818	0.313849
			2		0.001982	-0.001954
	0.5	1	0	0. 800423		
			1		0.366750	0.305089
			2		-0.066923	-0.190245
		2	0	0. 590971		
			1		0.287045	0.243832
			2		-0.016880	-0.103307
7 (water)	0.1	1	0	3.437454		
			1		0.726952	0.709948
			2		-1.983549	-2.017556
		2	0	2.745009		
			1		0.725246	0.712576
			2		-1.294516	-1.319857
	0.5	1	0	3.329354		
			1		0.639094	0.256583
			2		-2.051164	-2.816186
		2	0	2.653193		
			1		0.655773	0.371772
			2		-1.341646	-1.909647
100 (engine oil)	0.1	1	0	13.890674		
			1		-1.060918	-1.154713
			2		-16.012511	-16.200101
		2	0	11.283929		
			1		-0.485243	-0.559073
			2		-12.254415	-12.402077
	0.5	1	0	13.530921		
			1		-1.600134	-3.751520
			2		-16.731190	-21.033964
		2	0	10.987189		
			1		-0.896508	-2.586174
			2		-12.780206	-16.159538

heat transfer while the injection parameter decrease the flow. Meanwhile in Fig. 2 we can see the combined effects of Joule heating and viscous dissipation on the heat transfer rate. As we mentioned earlier, when $Ec = 0$, there are no effects of Joule and viscous heating. It can be observed that $-\theta'(0)$ reduced sharply with the increasing of *Ec*. Further, imposition of fluid suction has the tendency to increase the interface heat transfer. On the other hand, fluid injection tends to decrease the heat transfer rate at the wall but rapidly increase the rate as $Ec > 3$.

Fig. 3-5 show the effects of *Pr*, *Nr* and combined effects of Joule and viscous heating in the influence of suction and injection on the temperature profiles. The results indicate that the increasing of Prandtl number decrease the temperature profiles while the increasing of the values parameter *Nr* and *Ec* tend to enhance the temperature profiles.

Furthermore, the effects of magnetic parameter *M* on velocity profiles and temperature profiles are shown in Fig. 6 and Fig. 7, respectively. We noticed that increasing the parameter *M* will result in the reduction of velocity profiles. On the contrary, the temperature profiles increase as parameter *M* increase. In general, based on the results in Fig. 3-7, the suction parameter decreases the temperature profiles while injection parameter will increase the temperature profiles.

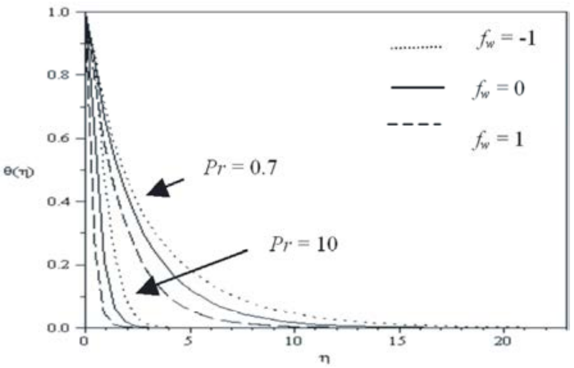

Fig. 3: Effects of Pr on temperature profiles when $Nr = 1$, $Ec = 1$, $M = 0.5$ with various values of f_w

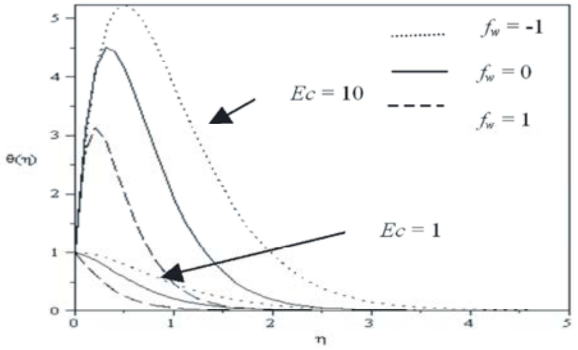

Fig. 5: Combined effects of Joule heating and viscous dissipation on temperature profiles when $Pr = 7$, $Nr = 1$, $M = 0.5$ with various values of f_w.

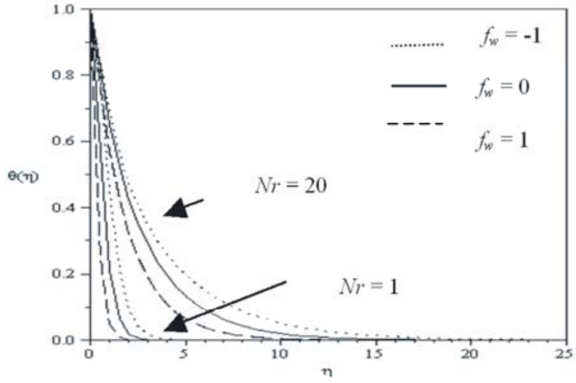

Fig. 4: Effects of Nr on temperature profiles when $Pr = 7$, $Ec = 1$, $M = 0.5$ with various values of f_w.

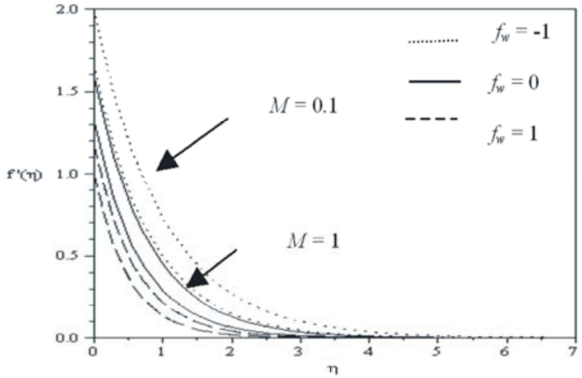

Fig. 6: Effects of M on velocity profiles when $Pr = 7$, $Nr = 1$, $Ec = 1$ with various values of f_w.

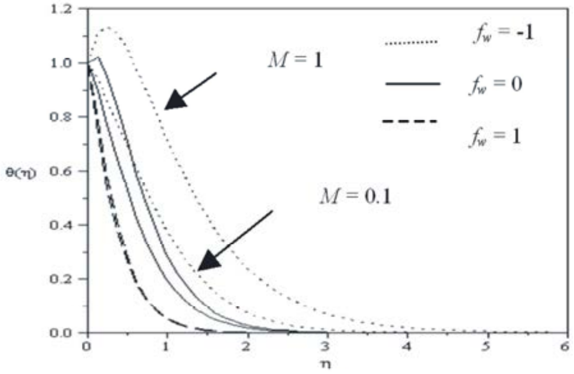

Fig. 7: Effects of M on temperature profiles when $Pr = 7$, $Nr = 1$, $Ec = 1$ with various values of f_w.

CONCLUSION

This paper presents the effects of radiation, Joule heating and viscous dissipation on MHD Marangoni convection boundary layer over a flat surface in the influence of fluid suction and injection. The general equations are converted into a system of self-similar equations by the similarity transformation technique. The effects of radiation parameter Nr, viscous dissipation Ec, Joule heating Ec. M, magnetic parameter M as well as suction and injection parameter are investigated and presented in tables and figures. Generally, when we increase the radiation parameter Nr, magnetic parameter M and combined effects of Joule and viscous heating, we can see the reduction in the interface heat transfer but the temperature profiles are increased. Not only that, the increased values of *M* tend to decrease the velocity profiles. Besides, it is seen that the boundary layer flows are greatly influenced by Prandtl number. As Pr increased, the surface temperature gradients also increased. However, the temperature profiles will decrease when Pr increased.

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