Harvesting vibration energy using piezoelectric material: Modeling, simulation and experimental verifications ☆

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ARTICLE INFO

Article history:
Received 16 November 2011
Accepted 29 October 2012
Available online 2 December 2012

Keywords:
Piezoelectric
Unimorph
Energy harvester
Frequency response function
Proof mass

ABSTRACT

Piezoelectric materials produce electrical charges when subjected to dynamic strain. These materials can be used to capture and store vibrational energy which later can be used to power up small devices. This paper presents an analytical estimation of voltage production of piezoelectric cantilever beam when subjected to base excitation, with and without attached proof masses. The beam is modeled using Euler–Bernoulli, also known as thin beam theory. As such, the model obtained here is applicable for micro- and nano-beams. The frequency response function (FRF) that relates the output voltage and transverse acceleration is identified for multi-mode vibration. These analytical predictions are then compared with experimental results and good agreement is obtained.

1. Introduction

Energy harvesting has been of interest of many researchers for decades. Researchers have put much effort into energy conversion [1] and recent researches has discovered that piezoelectric materials are one the best option in energy conversion from mechanical into electrical or vice versa. These materials are also widely used as sensors and actuators [2]. Many researchers have derived mathematical models for energy harvesting beam; most of them have used Euler–Bernoulli beam theory which is also known as thin beam theory. Models derived using this theory are applicable for both micro- and nano-beams.

The analytical solution for response of unimorph piezoelectric patch is presented by Erturk and Inman [3] with coupled solution involving small rotation. In the paper, the focus was on simulation analysis. Smits and Choi [4] have modeled a bimorph beam using the energy conservation method with various electrical and mechanical boundary conditions. However, no voltage generation has been presented in the paper.

Energy harvester researchers have done a lot of experiments to validate the analytical results. Authors such as DeVoe et al. [5] validated the model with experimental design but it was limited to actuator mode. The experiment on generated voltage across the load resistance was also demonstrated and the measurements were focused on the first natural frequencies [6,7]. Erick et al. have experimentally validated a unimorph membrane with piezoelectric element by varying the resistive load [8]. Validation for a bimorph cantilever with tip mass has been shown in the form of single mode solution at resonance excitation for series and parallel connection [9]. The efficiency of the system which is proportional to vibration frequencies were presented by Korla et al. [7]. The experimentation has been done with two different conditions, with and without proof masses. Therefore, two sets of the different natural frequencies were obtained.

This paper presents the distributed parameter solution of cantilever configuration for parallel connection of piezoceramic layers. The beam is connected in parallel as in the unimorph energy harvester. Details of the configuration are given in Section 3. The output voltage produced in response to the beam-to-translational base acceleration is due to the presence of the multi-mode solution.

The rest of the paper is organized as follows. Section 2 gives the mathematical model for a piezoelectric beam and an analytical estimation of the voltage output. Section 3 discusses the experimental setup to get validation between analytical and experimental results. Section 4 gives experimental results for the produced voltage, with and without the attached proof masses. These results are then compared with the analytical solution from Section 2. Section 5 gives the conclusion to this paper.

2. Mathematical model for piezoelectric beam

The model was recently presented by Fakhzan and Muthalif [10]. The analytical solution for the unimorph energy harvester is

☆ This work was done while the author was a master student sponsored by the Ministry of Higher Education, Malaysia under UniMAP Slab program.

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based on Euler–Bernoulli beam theory. The voltage obtained with response to the resistive load, R_l, as illustrated in Fig. 1.

The general equation of motion for a beam can be expressed as

$$\frac{\partial^2 M}{\partial x^2}(x,t) + \rho A(x)\frac{\partial^2 w}{\partial t^2}(x,t) = f_0(x,t) \tag{1}$$

where $M(x,t)$ is beam's bending moment, $\rho A(x)\frac{\partial^2 w}{\partial t^2}(x,t)$ is the inertia force acting on the beam where ρ = mass density, $A(x)$ is the cross sectional area and $\rho A(x)$ is the mass per unit length and $f_0(x,t)$ is the external force per-unit length applied to the beam. For base excitation, the external force is a function of time and is given as in the following equation:

$$f(t) = -\rho A(x)\frac{\partial^2 w_b(t)}{\partial t^2} \tag{2}$$

The cantilever beam is subjected to base displacement with the absolute displacement of the beam, $w(x,t)$ expressed in terms of the base displacement, $W_b(t)$ and the beam transverse displacement response to the base, $W_t(x,t)$:

$$w(x,t) = W_b(t) + W_t(x,t) \tag{3}$$

The constitutive equation, which relates the electrical and mechanical term for this system is

$$D_3 = d_{31}\sigma + \varepsilon_{33}^T E_3 \tag{4}$$

where D_3 is the electrical displacement, d_{31} is the piezoelectric strain constant, σ is the stress, ε_{33}^T is the permittivity at constant stress and E_3 is the applied electric component.

The eigen-function representing the n-th mode shape corresponding to the undamped free vibration problem is [10]

$$W_{tn}(x) = C_n[(\sin\beta_n x - \sinh\beta_n x) - \alpha(\cos\beta_n x - \cosh\beta_n x)] \tag{5}$$

where C_n is a constant and

$$\alpha = \frac{\sin\beta_n L + \sinh\beta_n L}{\cos\beta_n L + \cosh\beta_n L} \tag{6}$$

L is the length of the beam, and β_n's are the dimensionless frequency numbers obtain from the frequency equation given by

$$1 + \frac{1}{\cos\beta L\cosh\beta L} - R\beta L(\tan\beta L - \tanh\beta L) = 0, \tag{7}$$

where

$$R = \frac{m}{\rho A(x)L} \tag{8}$$

Eq. (6) is obtained using boundary conditions applied to the fixed and free end of the beam as given in [11]. The value of R, which determines the natural frequency of the beam, represents the ratio of the proof mass, m, over the beam mass, $\rho A(x)L$. As R increases, the natural frequencies decrease.

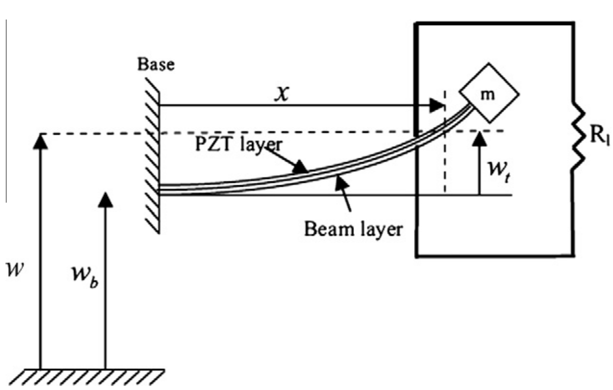

Fig. 1. Unimorph piezoelectric energy harvester with a tip mass excited under transverse base motions.

The n-th natural frequency, ω_n, is given as in the following equation:

$$\omega_n = (\beta_n L)^2\sqrt{\frac{EI(x)}{\rho A(x)L^4}} \tag{9}$$

The analytical solution for the output voltage is [10]

$$\left|\frac{V_0}{\omega^2 w_{b0}}\right| = \frac{\tau_c\omega\varphi_{tn}(t)\int_0^L W_{tn}(x)dx}{\sqrt{[\omega_n^2 - \omega^2(1+2\zeta_n\tau_c\omega_n)]^2 + [2\zeta_n\omega_n\omega + \tau_c\omega(\varphi_{tn}\frac{\vartheta}{\rho A} + \omega_n^2 - \omega^2)]^2}}, \tag{10}$$

where w_{b0} is the amplitude of the base translation, ζ_n is the beam damping ratio, and the time constant τ_c is defined as

$$\tau_c = \frac{R_l\varepsilon_{33}^T BL}{h_p}, \tag{11}$$

where R_l is the load resistance, B is the beam width and h_p is the piezoelectric layer thickness. The term φ_{tn} is given as

$$\begin{aligned}\varphi_{tn} &= -\frac{d_{31}E_p h_p(\frac{h}{2}+h_p)}{\varepsilon_{33}^T L}\int_{x=0}^{L}\frac{\partial^2 W_{tn}(x)}{\partial x^2}dx \\ &= -\frac{d_{31}E_p h_p(\frac{h}{2}+h_p)}{\varepsilon_{33}^T L}\left.\frac{dW_{tn}(x)}{dx}\right|_{X=L}\end{aligned} \tag{12}$$

where E_p is the piezoelectric Young's modulus, h is the beam layer thickness and W_{tn} is the transverse displacement of the beam and ϑ is given as

$$\vartheta = \frac{E_p}{2}Bd_{31}(h + h_p) \tag{13}$$

Simulation results using analytical solution derived in Eq. (10) for the beam without proof mass is shown in Fig. 2. The damping ratio used in Eq. (10) is estimated from experiment in order to get reasonable results.

3. Experimental setup

3.1. Experimental setup for unimorph cantilever beam with proof mass

The experimental setup to measure the voltage produced by PZT patch due to base acceleration is shown in Fig. 3. A piezoelectric patch model V21B from MIDE Technologies is used as the energy harvester.

The schematic diagram of the experimental setup is shown in Fig. 4. The experimental transfer functions (TF) are obtained using HP-Dynamic Signal Analyzer (DSA). The output of the system is the voltage produced from the piezoelectric cantilever beam (Ch 1) and the input to the system is the acceleration measured from the accelerometer (Ch 2). The transfer function obtained from Ch 1/Ch 2 gives Voltage/base acceleration. The V21B patch is made up from four main materials: glass-reinforced epoxy laminate (FR4), epoxy, piezoelectric material and espanex, where they are sandwiched together to form the energy harvester beam. The properties of the V21B are given in Table 1. Further details on the properties are given in Volture datasheet [11].

3.2. Important properties and setting

The internal impedance of the DSA is given in the technical specification as – 1 MΩ ± 10%. The impedance value is used for the resistive load, R_l when performing simulation studies. The accelerometer used has sensitivity of 100 mV/g.

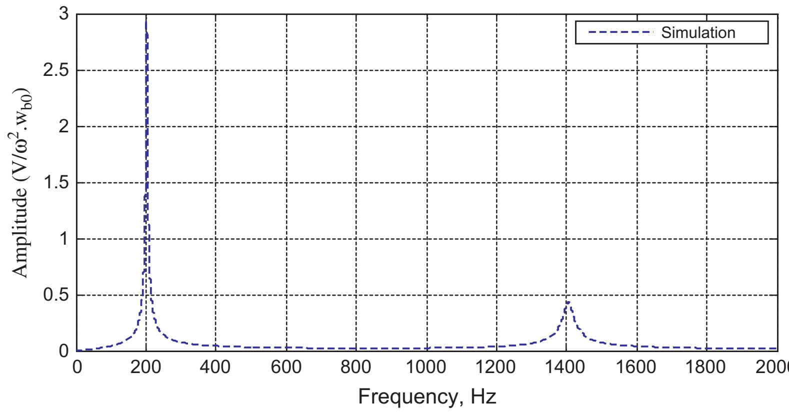

Fig. 2. Amplitude of simulated output voltage without proof mass.

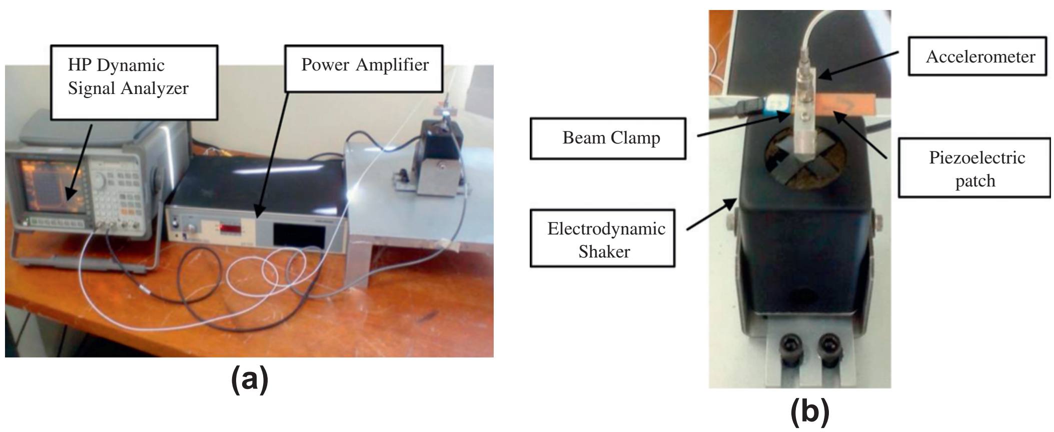

Fig. 3. (a) Experimental setup and (b) shaker with energy harvester beam.

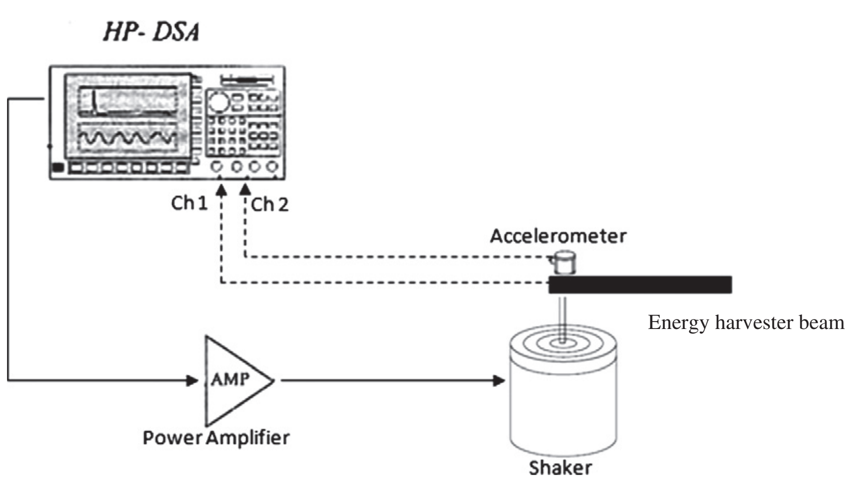

Fig. 4. Schematic diagram for experimental validation.

Table 1
The properties of V21B.

Dimensions of V21B		Piezoelectric properties		FR4 properties	
Length, L	0.06960 m	Coupling coefficient, k_{31}	0.36	Young's modulus, E_{FR4}	23.4 GN/m^2
Width, w	0.01702 m	Coupling coefficient, k_{33}	0.72	Poisson's ratio, υ_{FR4}	0.14
Thickness, h	0.000813 m	Density, ρ_{PZT}	7800 kg/m^3	Density, ρ_{FR4}	1.92 g/cm^3
Young's modulus, E	6.9×10^{10} N/m^2	Charge constant, d_{31}	-190×10^{-12} m/V		
Area moment of inertia, I	1.5990E−06 m^4	Voltage constant, g_{31}	-11.3×10^{-03} Vm/N		
Area, A	1.3832E−05 m^2	Density, ρ	7800 kg/m^3		

Table 2
Estimated zeta values using half-power bandwidth method.

Properties	1st natural frequency				2nd natural frequency			
	ω_l (rad/s)	ω_n (rad/s)	ω_u (rad/s)	Zeta, ζ	ω_l (rad/s)	ω_n (rad/s)	ω_u (rad/s)	Zeta, ζ
No mass	199.4	208.1	205.2	0.0211	1382	1403	1390	0.0075
2.4 g	103.2	113.2	110.6	0.0452	938.3	1060	1012	0.0601
4.8 g	77.41	82.71	80.68	0.0328	741.7	824.9	782.9	0.0531
7.2 g	61.07	79.46	75.7	0.1214	576.55	689.33	618.5	0.0911

3.3. The damping ratio, ζ_n, of the piezoelectric beam

Modal damping ratio is estimated from the frequency response function curve using the half-power bandwidth method. These estimated damping ratios are used for analytical simulation in Section 2 above. The half power bandwidth equation for $\zeta \ll 1$ is given in Eq. (14) [12]

$$\frac{\omega_l - \omega_u}{\omega_n} \cong 2\zeta, \tag{14}$$

where ω_l, ω_u are the lower and upper frequencies when magnitude of FRF is $|V/A| = |V/A|_{max}/\mathrm{sqrt}(2)$. The calculated zeta value for the 1st natural frequency is different from the 2nd natural frequency of the beam. It is important to note that the zeta value changes when the proof mass is added. The different amplitude of output voltage will contribute to the zeta value where the differences between ω_l and ω_u will differ from one to another natural frequency of the beam. The estimated values of zeta for the first and second natural frequencies with different proof mass are given in Table 2.

4. Results and discussion

The experiment was carried out using four different proof masses; 0, 2.4 g, 4.8 g and 7.2 g. Fig. 5 shows the voltage generated for the first two modes with different proof masses. From Fig. 5, it can be observed that the voltage produced at the first mode increases with the weight of the proof mass. Also, the first natural frequency decreases as the weight of the proof mass increased. Hence, the proof mass has the ability to tune the natural frequency of the beam. The experimental results are then compared with the

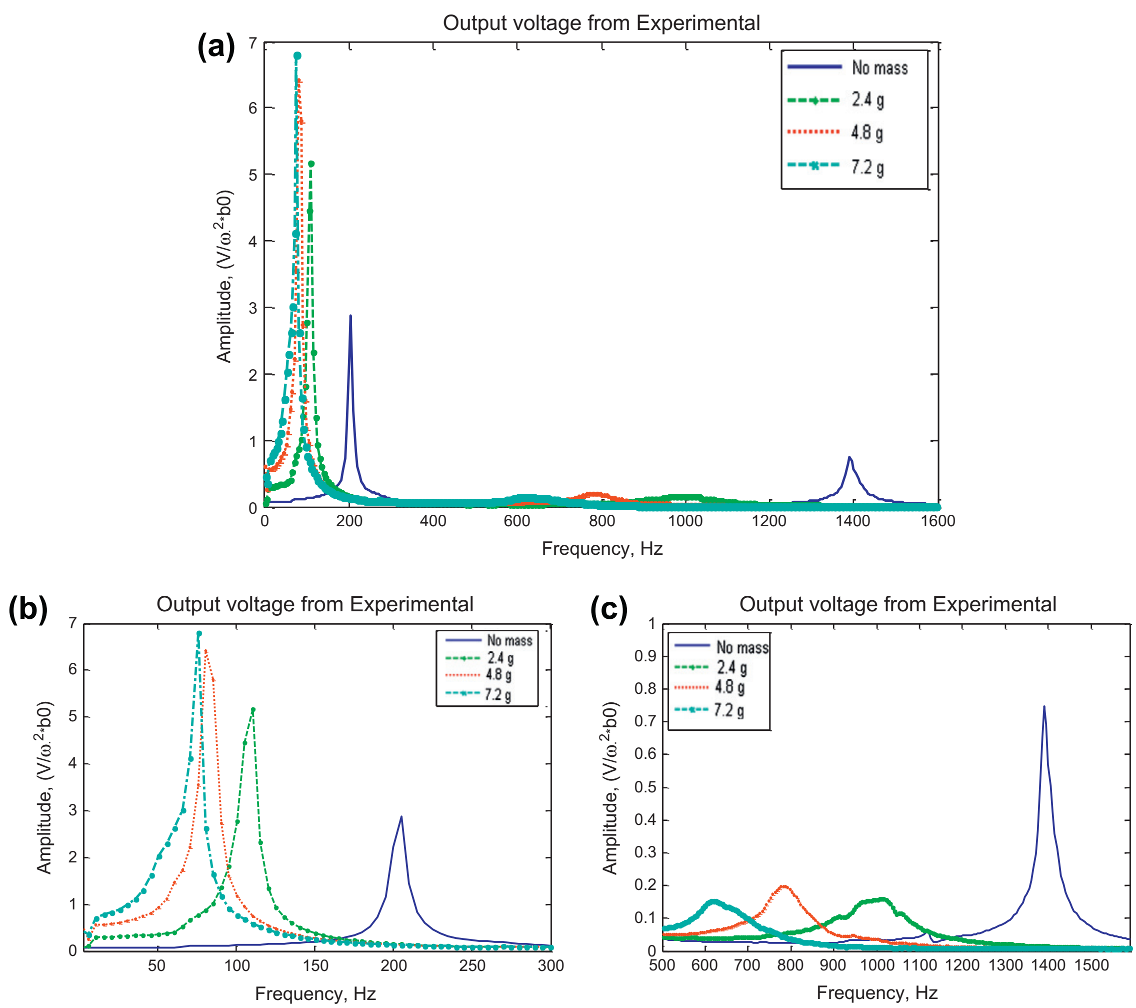

Fig. 5. (a) Experimental voltage of cantilever for difference proof mass at the tip end for 1st and 2nd natural frequencies, (b) close up for 1st natural frequency, and (c) close up for 2nd natural frequency.

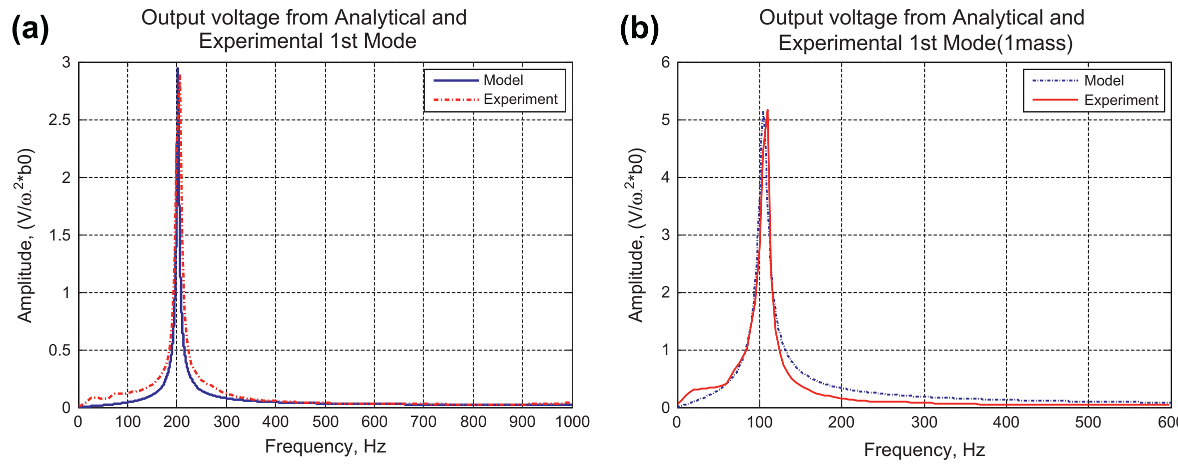

Fig. 6. Comparison of output voltage of analytical and experimental for the 1st natural frequency: (a) with no mass attached and (b) with 1 mass attached.

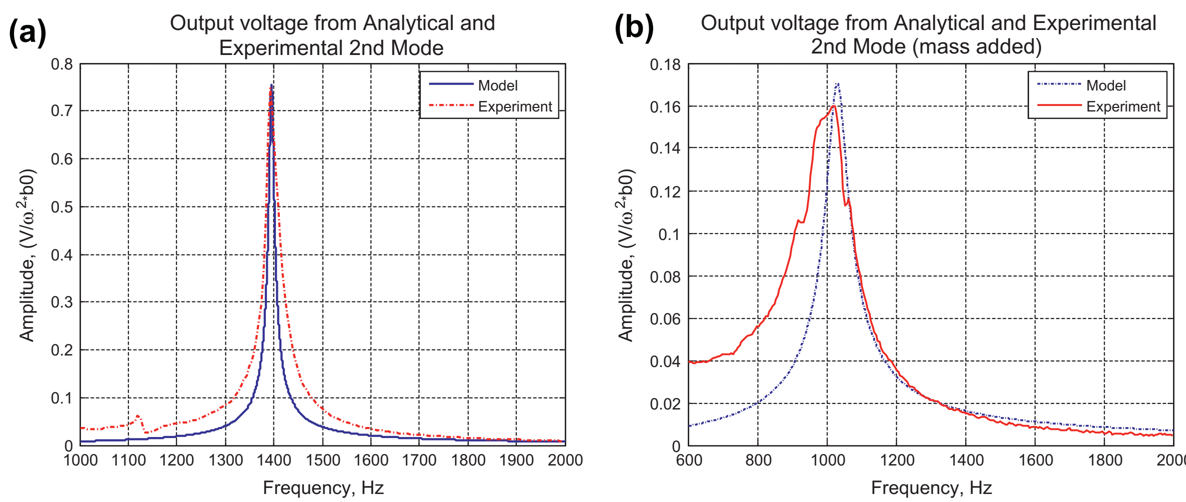

Fig. 7. Comparison between analytical and experimental voltage for the 2nd natural frequency: (a) with no proof mass and (b) with 2.4 g proof mass attached.

analytical simulation. The correction factor is taken from the impedance of the HP dynamic analyzer; in which this value will compensate the error between the simulation and experimental results. The correction factor used in the simulation is 1.55×10^4. The comparisons between the analytical and experimental results are shown in Fig. 6a and b for the first and the second mode. From Fig. 6a, the output voltage obtained experimentally and analytically for first natural frequency, $\omega_1 = 200$ Hz, with no mass attached to the beam is 2.88 V and 2.94 V, respectively, where the percentage difference is 2.04%. In Fig. 6b, a proof mass of 2.4 g is added, which resulted to the natural frequency decreasing to 110.56 Hz while the output voltage increases to 5.17 V. The estimated voltage from analytical solution is 4.99 V which makes the percentage difference 3.48%.

The results for 2nd natural frequency with no proof-mass and with proof mass of 2.4 g are shown in Fig. 7a and b respectively. The output voltage obtained experimentally for the 2nd mode decreases from 0.747 V to 0.161 V when a proof mass is added. Similarly, the output voltage from simulation shows reduction from 0.754 V to 0.171 V using the same proof mass. The damping factor for the 2nd mode is 0.00755, which is smaller than the 1st mode damping factor. Fig. 7b shows some variations between the experimental and simulation results. However, the difference between the two peaks at 600 Hz is not significant; 0.03 V and continue to decrease to 0.01 V when it reaches the peak at 1020 Hz. The peak differences between experimental and simulation results are 0.93% and 5.85% for Fig. 7a and b, respectively. The increasing damping factor when the mass increases is caused by the Teflon tape used to attach the proof mass to the end of the beam which added some extra mass and stiffness to the system. This disturbance to the system restricts the end of the beam from fully deflect hence causing error to the result.

In Figs. 6a and b and 7a and b, some fluctuations or ripples can be seen in experimental results before reaching the peak. This could be from the influence of other modes such as torsion in which the beam could behave as a plate [3].

5. Conclusions

For years, piezoelectric material has been used to be the medium of conversion from ambient vibrational energy into electrical energy. The harvested energy can be used to energize electrical devices or stored for later usage. Battery is known as a finite storage system that can be improved using energy harvester technology. Furthermore, the charging time of the battery can be shortened where simultaneous charging can be done together with electrical generator. Therefore, it is important to acknowledge the voltage or power produced by the energy harvester system.

The analytical solution for a cantilever beam with proof mass attached at the beam tip is discussed in this paper. Thin beam theory and piezoelectric constitutive equations are used to solve the energy harvester model to give the output voltage for the first two modes, which is the scope of this paper.

From the work done, small discrepancies are obtained between simulation and experimental results. The effect of using different proof masses which is attached at the end of the cantilever beam is also discussed. It is found that for heavier proof mass, the value of natural frequency decreases and the value difference between its subsequent natural frequencies is smaller. In addition, Eqs. (6)–(8) can be used to determine the bandwidth of the energy harvester and to choose the required proof mass. From the experiments, it is determined that the energy harvester operates effectively at low working bandwidth <1 kHz, therefore an appropriate proof mass is needed to tune the natural frequency so that it falls within this bandwidth range.

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