



IMPACT MODELS AND COEFFICIENT OF RESTITUTION: A REVIEW

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ABSTRACT

Impact is a complicated phenomenon that occurs when two or more bodies collide at a short time period. Impact model is used to determine the responses of the contact bodies due to the impact event. Furthermore, energy loss due to impact can be determined by coefficient of restitution (COR). Although numerous impact models have been reported in the literatures, the works to improve the model are continuously explored to achieve perfection of the impact model. The aims of this paper are to present the methodologies that have been used to obtain the impact models and COR, evaluate on the pros and cons of the previous impact models and determine the potential area to improve the current impact models and COR. The methods to obtain COR from experiment and finite element method (FEM) are briefly discussed. Besides that, several significant impact models developed by the researchers are also compared. It is found that more works to determine COR in oblique and repeated impacts should be performed. Until now, the viscoplastic impact model is considered to be the most reliable in impact application. This review is intended to assist the derivation of impact models in the future that can improve the accuracy and solve the problem in the previous impact models to be applied in various impact applications.

Keywords: impact model, coefficient of restitution, contact law.

INTRODUCTION

Impact is defined as the collision between two bodies at an instant of time (Stronge, 2004). During impact, the bodies experience a high force level at very brief duration, while energy is rapidly dissipated and high acceleration and deceleration occurred (Gilardi and Sharf, 2002). Two phases happen during impact; the compression phase occur when the bodies initially start to contact and compressed against each other and the restitution phase takes place when the bodies start to separate but still in contact (Gharib and Hurmuzlu, 2012). The latter phase ends when the bodies are completely separated.

Impact model is theoretically derived based on the compression and restitution phases. In general, the impact model can be divided into several types; perfectly elastic (no energy loss), partially elastic (energy loss with no permanent deformation), perfectly plastic (all energy loss with permanent deformation) and partially plastic (some energy loss with permanent deformation). A linear impact model is developed by a compliance of linear stiffness or damping element. On the other hand, the nonlinear impact model is developed by a compliance of nonlinear stiffness or damping element. The addition of damping element (also known as viscous element) in the compliance model causes the impacted bodies to be deformed in both elastic and viscous characteristics and it is known as viscoelastic or viscoplastic models respectively.

The objective of developing the impact model is to obtain the response of the bodies during the post-impact, based on the known parameter during the pre-impact (Gilardi and Sharf, 2002). However, since the

impact model depends on many parameters, the work to obtain the final solution is very complex. To overcome this problem, coefficient of restitution is used to get the solution in the impact model.

Coefficient of restitution, COR, e , is a parameter used to determine the energy loss during impact. The value of e is in the range $0 \leq e \leq 1$, where ($e = 0$) indicates a perfectly plastic collision (total energy loss) while ($e = 1$) indicates a perfectly elastic collision (no energy loss). However, in practices, it is quite impossible to create a perfectly elastic collision because some of energy is dissipated to sound, heat, strain energy and etc. COR value is depends on the impact speed (linear and angular movements), material properties, geometry, sliding friction and contact period. The first COR model was developed by Newton (kinematic), followed by Poisson (kinetic) and finally by Stronge (energetic).

$$\text{Kinematic COR: } e = -\frac{v_f}{v_0} \quad (1)$$

$$\text{Kinetic COR: } e = -\frac{p_f - p_c}{p_c} \quad (2)$$

$$\text{Energetic COR: } e = -\frac{W_r}{W_c} \quad (3)$$

where v_f and v_0 are the final relative velocity and initial relative velocity respectively, p_f and p_c are the



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normal impulse for restitution and compression respectively and W_r and W_c are the work done during restitution and compression phases, respectively. These definitions of COR are equivalent, unless if the configuration of impact involve with friction and the direction of the slip changes during collision, only the energetic COR can provides the correct solution (Ismail and Stronge, 2008). The impact models and COR are widely used to solve the problems in sports engineering (Cross, 2010), (Goodwill and Haake, 2001), (Cross, 2014), geology (Ashayer, 2007), (Imre et al., 2008), coal gasification industry (Gibson et al., 2013), automotive (Batista, 2006), robotic (Vasilopoulos et al., 2014) and many more. Previously, extensive studies have been reported on development and modification of the impact models. However, these impact models have their own limitation and the solution in impact models are not usually straightforward. To address this concern, many researchers are still working to create and modify the impact models in order to improve the current impact models (Jacobs and Waldron, 2015), (Rathbone et al., 2015), (Alves et al., 2015). The aims of this paper are to present the methodologies that have been used to obtain the impact models and COR, discuss on the pros and cons of the previous impact models and determine the potential area to improve the current impact models and COR.

PREVIOUS STUDIES OF COR

Previous researchers use experiment and finite element analysis (FEA) to obtain the value of COR in order to solve the analytical model of impact models. Three types of impact are always performed; direct, oblique and repeated impacts.

Several types of balls had been vertically drop on a brass rod by Cross (Cross, 2010) to obtain the change of speed during impact and dynamic hysteresis curve at low speed of impact was presented. Even at low impact speed, the local damage and plastic deformation were still experienced by the ball, where the similar result was obtained by Johnson (Johnson, 1985). Furthermore, the energy loss on the ball was greater in a dynamic analysis compared to the static analysis.

Besides that, Aryaei et al. (Aryaei et al., 2010) studied the effect of ball size to the COR value by drop test experiment and FEA. They found that the ball size gave a significant effect to the COR result especially when the ball and the impacted surface were made of different materials. However, this result is contradict with the theoretical expression by Stronge (Stronge, 2004) where the ball size did not effect on the COR result in direct impact problems.

Most of researchers used a sphere shape as the object for impact but only a few studies analysed the COR by using the other shapes. Buzzi et al. (Buzzi et al., 2012) experimentally compared the result of kinematic COR for circular, elliptic, square and pentagonal shapes of blocks. These blocks were dropped with initial spin through the ramp and impacting a rigid landing block in an oblique

direction. Besides that, the measurement of COR for the irregular shaped particle had been experimentally performed by several studies (Hastie, 2013), (Li et al., 2004). A high speed camera and a mirror were used to obtain the particle's velocity in three dimension and the particle was assumed as sphere and cylinder shape for the analytical solution (Hastie, 2013).

For the analysis in oblique impact, normal and tangential COR were considered due to change of impact direction in two different axes. Cross (Cross, 2002) used tennis ball and superball to impact the wood, emery and rebound ace surfaces in oblique direction. As a result, the angle of incidence affect greatly to the COR. Furthermore, negative value of tangential COR could be obtained due to low angle of incidence and coefficient of sliding friction that encourage the ball to slide throughout the impact. Besides that, Dong and Moys (Dong and Moys, 2006) performed the oblique impact with initial spin (clockwise and counter-clockwise direction) was introduced on the steel ball. They found that at low incidence angle, the value of horizontal COR could be negative or more than one due to the angular direction of the spinning balls.

Minamoto and Kawamura (Minamoto and Kawamura, 2011) performed a high speed impact (up to 20 m/s) between two steel spheres by using air gun in experiment and it was validated by FEA. According to their results, COR reduces when the impact speed is increased, however, the rate of reducing COR is larger in low impact velocity (below 2.5 m/s).

During impact, the kinetic energy is dissipated by plastic deformation and stress wave propagation. For the partially elastic impact, the kinetic energy is dissipated by the propagation of elastic wave (Mangwandi et al., 2007). Hunter (Hunter, 1957) found that the energy loss due to elastic wave propagation is less than 1% of the initial kinetic energy for a steel ball impacting a large steel or glass. Besides that, Wu et al. (Wu et al., 2005) performed the impact of elastic sphere with the elastic and elastic-perfectly plastic substrates by using FEA. They found that the energy dissipation due to wave propagation is less than 3% of the kinetic energy and it depends on the size of the impacted surface. Moreover, the kinetic energy loss due to wave propagation is higher when the impact involve with slender bodies such as rods, beams, plates and shells (Seifried et al., 2005).

The study of COR for repeated impacts between two bodies is still given less attention by the previous researchers. Seifried et al. (Seifried et al., 2005) performed repeated impact between steel sphere and aluminium rod by using experiment and FEA. They found that when the number of impact is increased, the impact force and COR are also increase due to residual stress that reduce the energy loss. Furthermore, Minamoto et al. (Minamoto et al., 2014) also obtain the similar results for the impact developed by viscoplastic model in FEA.



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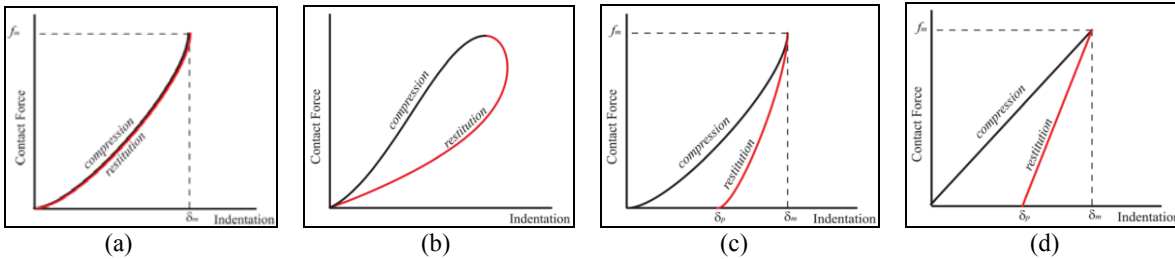


Figure-1: Force-indentation curve for (a) Nonlinear elastic model, (b) Nonlinear viscoelastic model, (c) Nonlinear elastoplastic model and (d) Linear viscoplastic model. Adapted from Gharib and Hurmuzlu, 2012.

EVOLUTION OF IMPACT MODEL

Impact between two rigid bodies can be treated by using discrete or continuous model. In discrete model, the velocity is instantaneously changed during the impact and it always measured by the impulse-momentum method. Meanwhile, the continuous model represent the collision in a finite duration. The force-indentation relationship during impact could be measured by using the continuous model, so the impact behaviour could be analysed. The evolution of the discrete and continuous models will be discussed in the next sections.

Discrete model

Impulse-momentum principle is a discrete model that was conventionally used to solve the problem in impact. This principle assumed that the impact is happen at very brief of period and there is no change in the configuration of the impacting bodies during the impact. The analysis is divided into two intervals, which are during pre and post impact. COR or the impulse ratio that was obtained from the experiment is employed into this impulse-momentum principle in order to determine the post impact velocity and energy loss during impact. In general, the conservation of momentum is applied in the impulse-momentum principle, which is given by:

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2 \quad (4)$$

where m_1 and m_2 are the mass of the first and second objects respectively, while u_1 and u_2 are the initial velocity and v_1 and v_2 are the final velocity of the first and second objects respectively. However, the results obtain by this principle is inconsistent when there is a friction on the bodies during impact and COR value is difficult to be determined for the impact between the flexible bodies (Wang and Mason, 1992), (Escalona et al., 1998).

Continuous model

Continuous model, also known as compliance based method is proposed to overcome the problems in the discrete model. During collision between two bodies, the compliance of a small contact region around the initial contact point is represented by the lumped parameter, which are stiffness and damping elements. As the contact force between the bodies is continuously act during impact, so the force-indentation profile is the primary

output in this continuous model. It could be derived from the equation of motions that was developed from the compliance at the contact point.

The simplest elastic impact model is expressed by Hooke's Law. This linear perfectly elastic model is represented by a linear spring element in which the spring embodies the elasticity of the contacting surfaces and the force is expressed by:

$$F = \kappa \delta \quad (5)$$

where κ is the stiffness of the spring and δ represents the relative indentation between the impacting bodies.

Besides that, Hertz (Hertz et al., 1896) proposed a nonlinear perfectly elastic contact between two isotropic spheres and the force-indentation curve is shown in Figure-1(a). A nonlinear spring element is represented as a compliance at the contact point and the impact force, F is given by:

$$F = \kappa \delta^n \quad (6)$$

where n is the nonlinear power exponent. This is a nonlinear perfectly elastic model where the application is limited to low impact velocity and only applicable in elastic deformation and hard bodies. For most impact cases, the initial kinetic energy is dissipated due to wave propagation, plastic deformation or other factors. However, Hooke's law and Hertz model are not account for energy dissipation, thus the COR is unity.

Because of that, damping element (dashpot) is added into the Hertz model to account for energy dissipation during impact. The combination of the stiffness and damping elements is called viscoelastic constitutive model and it is limited to low range of relative impact velocity. Furthermore, this model is only valid for elastic deformation as similar with Hertz model, thus, the indentation value return to zero at the end of impact as shown in Figure-1(b).

Kelvin-Voigt model is the first linear viscoelastic impact model that consists of a linear spring and a linear dashpot element connected in parallel configuration as shown in Figure-2(a).



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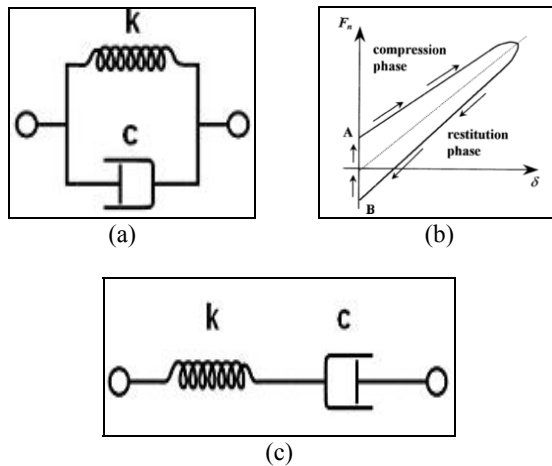


Figure-2: (a) Kelvin-Voigt model, (b) Force-indentation curve for Kelvin-Voigt model and (c) Maxwell model. Adapted from Gilardi and Sharf, 2002 and Vladimír, 2010.

This model is a linear viscoelastic and rate-dependent impact model, where the impact force is defined as:

$$F = \kappa\delta + c\dot{\delta} \quad (7)$$

where c and $\dot{\delta}$ are the damping coefficient and rate of indentation between the impacting bodies respectively. However, according to the force-indentation relationship in Figure-2(b), the contact force is nonzero at initial and final impact due to presence of damping element.

To encounter this problem, Hunt and Crossley (Hunt and Crossley, 1975) proposed a nonlinear viscoelastic impact model based on the Hertz and Kelvin-Voigt models. They replace the linear spring and damping elements in the Kelvin-Voigt model with nonlinear spring and damping elements, thus the impact force is expressed by:

$$F = \kappa\delta^n + c\delta^n\dot{\delta} \quad (8)$$

Throughout this nonlinear viscoelastic model, the force at the beginning and final are zero, which successfully solve the problem in Kelvin-Voigt model. In addition, the damping coefficient can be expressed as a function of the coefficient of friction since both are related to energy dissipation parameter during impact (Gilardi and Sharf, 2002). Thus, many researchers had proposed the expression of the damping coefficient and the nonlinear power exponent, n through various experimental and theoretical approaches, which had been compared by Alves et al. (Alves et al., 2015). However, Yigit et al. (Yigit et al., 2011) claims that this model is only considered in a particular problem and it does not depend on any physical explanation. In addition, the solution is more complex due to additional of nonlinear parameter

into the Hertz model (Yigit et al., 2011). Furthermore, the contact parameters (material properties and geometric condition) are difficult to determine when using a nonlinear damping model, so these parameters are usually adjusted based on the experimental results (Gilardi and Sharf, 2002).

In contrast with Kelvin-Voigt model, Maxwell model is a linear viscoelastic constitutive model that combining a nonlinear spring and damping elements in series compliance as shown in Figure-2(c). Hence, the impact force is defined by:

$$F = \kappa\delta = c\dot{\delta} \quad (9)$$

During impact, the normal force is smoothly increases with normal compression on the deforming region and the same force is act in both spring and damping element (Stronge, 2004). Maxwell model is differ from the Kelvin-Voigt model, where the force at the beginning and final impact is equal to zero. Nonetheless, this model results in COR that is independent of impact velocity, which deny the experimental and analytical evidences that COR is dependent to impact velocity (Stronge, 2004), (Yigit et al., 2011).

The previous impact models (elastic and viscoelastic) are only applicable for very low impact velocity where the material deformation is in elastic region. However, most of the impact practice are resulting the bodies to deform in both elastic and plastic region. For example, Johnson (Johnson, 1985) performed an impact experiment between two metallic bodies and found that the contact stress is high enough to cause material yielding and plastic deformation even at impact velocity of 0.14 m/s.

Hence, numerous researchers had developed elastoplastic impact models that include plastic or permanent deformation of the materials (Yigit and Christoforou, 1994), (Thornton, 2013), (Lim and Stronge, 1999), (Thornton et al., 2013), (Christoforou, 1993). Goldsmith (Goldsmith, 1960) proposed a nonlinear elastoplastic model based on the Hertz model that can represent the plastic deformation during the restitution phase as shown in Figure-1(c) which is expressed by:

$$F = F_{\max} \left(\frac{\delta - \delta_p}{\delta_{\max} - \delta_p} \right)^n \quad (10)$$

where F_{\max} and δ_{\max} are the maximum normal force and indentation during compression phase and δ_p is the permanent indentation after separation. Then, Lankarani and Nikravesh (Lankarani and Nikravesh, 1994) provide an easier solution to determine δ_{\max} and δ_p . On the other hand, Yigit and Christoforou (Yigit and Christoforou, 1994) proposed a nonlinear elastoplastic contact law based on the Hertz and Johnson impact models. They divided the impact event into three phases.



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At first, the elastic loading is considered as Hertzian contact, followed by elastic-plastic loading until a yield point and lastly is the Hertzian elastic unloading where the permanent deformation is considered. In their elastoplastic model, the parameters are easily obtained from material properties and contact geometry and this model also account with the permanent deformation. However, the energy loss due to wave propagation is not considered in this elastoplastic model (Yigit et al., 2011).

To encounter this problem, Ismail and Stronge (Ismail and Stronge, 2008) proposed a linear viscoplastic impact model, based on the Maxwell model. The force-indentation curve for this viscoplastic model is shown in Figure-1(d). They replaced the elastic element (linear spring) with an elastoplastic element (bilinear spring) and connects them with a linear damping element in series. Thus, the energy loss due to plastic deformation, wave propagation and other factors are accounted in this viscoplastic impact model. Furthermore, this model can be solved by analytical and numerical analyses while the elastoplastic impact model that was proposed by Yigit and Christoforou (Yigit and Christoforou, 1994) can only be solved by numerical analysis. Nonetheless, as similar with the Maxwell model, this model results in COR that is independent of impact velocity, which deny the experimental and mathematical evidence that COR is dependent to impact velocity (Stronge, 2004), (Yigit et al., 2011).

So, in 2011, Yigit et al. (Yigit et al., 2011) improve the plastic loss factor coefficient in the Ismail viscoplastic model that imposed the COR to be dependence on the impact velocity. Furthermore, they also developed a nonlinear viscoplastic model according to their previous elastoplastic impact model and obtained an almost similar results with Ismail. The disadvantage of this model is observed in the force-indentation curve for elastic impact (at impact velocity of 0.1 m/s), where there is nonzero indentation value at the end of impact. This is happen due to permanent deformation of damping element and there is no elastic element parallel to the damping element (Yigit et al., 2011). Thus, this viscoplastic model can be improved by adding a spring element parallel to the damping element as shown in Figure-3.

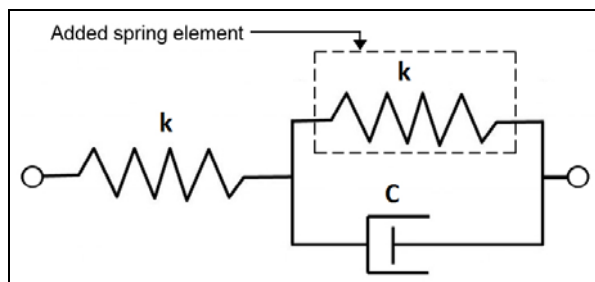


Figure-3: Addition of spring element parallel to the damping element in viscoplastic model.

Through the addition of this spring element, the deformation in the elastic impact could be restored at the end of impact. This model is a combination of Maxwell and Kelvin-Voigt model and it is similar to the standard solid model. However, the used of this model for the viscoplastic impact has not been explored by any researchers until now.

CONCLUSIONS AND RECOMMENDATIONS

Review on the coefficient of restitution (COR) and impact models have been presented in this paper. Until now, energetic COR that was introduced by Stronge is the most consistent and applicable in a wider application. The experimental works to determine COR in oblique and repeated impacts are still given less attention by the previous researchers. Although the analysis of impact is easier by using numerous simulation software that available today, however, the accuracy of this numerical analysis actually depends on the theory of the impact model itself. This is the reason of researchers are still developing and modifying the impact models until today. Throughout the previous discussions, every impact model has their own advantage and disadvantages. Until now, the viscoplastic impact model is claimed to be the most reliable in impact application. For the future work, it is worth to improve the current viscoplastic impact model as this model still has a certain limitation. It could be done by developing a viscoplastic impact model based on the standard solid model. This model is hypothetically valid in both elastic and plastic impact that has not been accurately predicted by the previous impact models. However, the validity of this proposed model has to be experimentally and numerically verified in the forthcoming works.

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