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FUZZY LINEAR PROGRAMMING: A MODERN TOOL FOR DECISION MAKING

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Abstract. The modern trend in industrial application problem deserves modeling of all relevant vague or fuzzy information involved in a real decision making problem. In the first part of the paper, some explanations on tri partite fuzzy linear programming approach and its importance have been given. In the second part, the usefulness of the proposed S-curve membershi p function is established using a real life industrial production planning of a chocolate manufacturing unit. The unit produces 8 products using 8 raw materials, mixed in various proportions by 9 different processes under 29 constraints. A solution to this problem establishes the usefulness of the suggested membershi p function for decision making in industrial production planning.

Key words: Fuzzy linear programming, Satisfactory solution, Decision maker, Implementer, Analyst, Fuzzy constraint, Vagueness.

1.0 INTRODUCTION

In this paper, the theory of a tripartite interactive Fuzzy Linear Programming (FLP) is proposed and an application of this proposal in solving for a profit function in an industrial production planning problem is suggested. The tri partite FLP gets required data inputs from the analyst, the decision maker and the implementer. The data and the contraints are fuzzy in their characteristics. The entire problem of profit optimization is solved in an interactive way among the analyst, decision maker and the implementer. In the tripartite planning, the author is the analyst who gets data from the other two and searches for a solution in an effort to satisfy them. Now a days, it is almost impossible to achieve a successful development without these three participants in the interactive decision making process.

Decision making is possibly the most important and inevitable aspect of application of mathematical methods in various fields of human activity. In real-world situations, decisions are fuzzy, at least partly. The first step of attempting a practical decision-making problem consists of formulating a suitable mathematical model of a system

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or a situation to be analyzed. Moreover, if we intend to make reasonably adequate mathematical models of such situation, we should be able to introduce fuzziness into our models and to suggest means of processing fuzzy information [1,2]. Fuzzy logic theory and applications have a vast literature. With regards to documented literature, we can classify the development in fuzzy theory and applications as having three phases; phase 1 (1965-1977) can be referred to as academic phase in which the concept of fuzzy theory has been discussed in depth and accepted as a useful tool for decision making. The phase 2 (1978-1988) can be called as transformation phase whereby significant advances in fuzzy set theory and a few applications were developed. The period from 1989 onwards can be the phase 3, the fuzzy boom period, in which tremendous application problems in industrial and business are being tackled with remarkable success.

Currently fuzzy technique is very much applied in the field of decision making. Fuzzy methods have been developed in virtually all branches of decision making, including multi-objective, multi-person, and multi-stage decision making. Apart from these, other research work connected to fuzzy decision making are applications of fuzzy theory in management, business and operation research [3]. In several such applications, linear membership functions such as triangular, and trapezoidal have been used. The details about logistic functions that can be built on a non linear membership function have been reported recently [4]. A non linear membership function has been used in FLP problems to encourage an interaction among decision makers to achieve a satisfactory solution.

In the applications using FLP, it is often difficult to determine the coefficients of the problem with precision because they are either specified subjectively by the decision maker or they are given through procedures requiring subjective answers to questions posed by analysts. So, to deal with imprecise data, fuzzy intervals may be defined where coefficients of the criteria are given by intervals [5].

Many problems in science and engineering have been considered from optimization point of view. As the environment is much influenced by the disturbance of social and economical situations, optimization approach is not always the best. It is because, under such turbulence, many problems are ill-structured. Therefore, a satisfactory approach may be better than that of optimization. Here, the procedure is described as how to deal with decision problems that are described by FLP models and formulated with elements of imprecision and uncertainty. In this respect it is necessary to study FLP models in which the parameters are partially known with some degree of precision.

Decision-making in the real world problems whose structures are not known exactly is our concern. For instance, the structure of a crisp linear programming problem is defined by its parameter set, the matrix A the vectors B and C. There are many cases when A, B and C cannot be precisely given. To deal quantitatively with imprecision, one can use the concepts and techniques of fuzzy set theory. Therefore the initial optimization problem has to be reformulated and a solving procedure must be found. Most of the conventional procedures do not tackle the problem in real life. They implicitly assume that the structure is fixed. In fact, the decision-maker needs flexible or robust models in order to face evaluative situations [6]. Fuzzy constraints or fuzzy parameters can reflect the capacity to absorb changes encountered in real life.

2.0 THE INTERACTIVE FUZZY LINEAR PROGRAMMING (IFLP) PROBLEMS

In the past, studies on IFLP problems were considered on the bi partite relationshi p of the decision maker and analyst [7]. This is with the assumption that the implementers are a group "robots" that are programmed to follow instructions from the decision maker. This notion is now a history. One has to consider the tripartite relationship (Figure 1) where decision maker, analyst and implementer will interact in finding fuzzy satisfactory solution in any given fuzzy system. This is because the implementers are human being and they have to accept the solutions given by decision maker and the analyst to be implemented under turbulent environment.

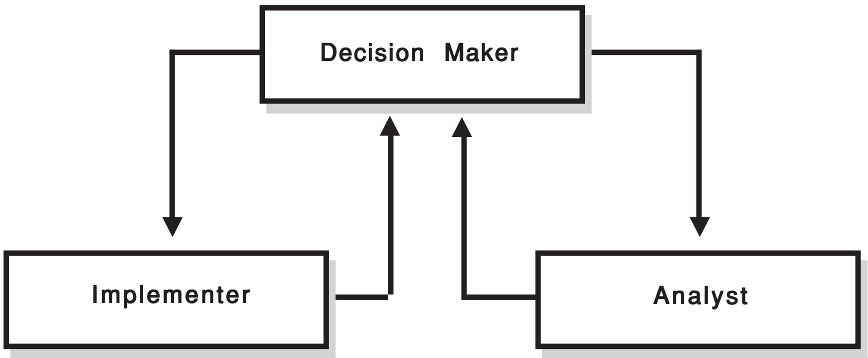

Figure 1 The block diagram for the tri partite fuzzy system

First the decision maker will communicate and describe the fuzzy problem with the analyst. Based on the data that are provided by the decision maker, the analyst will formulate membershi p functions, solve the fuzzy problems and provide the solution back to the decision maker. After that, the decision maker will provide the fuzzy solution with a trade-off to the implementer for implementation. Here again the implementer has to interact with decision maker to obtain an efficient and highly productive fuzzy solution with a degree of satisfaction. This fuzzy system eventually will be called as high productive fuzzy system. This is the concept emphasized in this paper.

3.0 FUZZY OPTIMIZATION PROBLEM

Due to limitations in resources for manufacturing a product and the need to satisfy certain conditions in manufacturing and demand, a problem of fuzziness occurs in industrial systems. This problem occurs also in chocolate manufacturing when deciding a mixed selection of raw materials to produce varieties of products. This is referred here to as the Product- mix Selection Problem [7]. The data for this problem are taken from the data-bank of Chocoman Inc, USA [7]. Chocoman produces varieties of chocolate bars, candy and wafer using a number of raw materials and processes. The objective of this paper is to use a S-shaped nonlinear membership function for obtaining a profit maximization procedure through Interactive FLP (IFLP).

4.0 DISCUSSION OF S- SHAPED MEMBERSHIP FUNCTION

The S-curve membership function is a particular case of the logistic function with specific values of B, C and α. These values are to be found out. This logistic function as given by Equation 1 and depicted in Figure 2 is indicated as S-shaped membership function [8,9].

We define, here, a modified S-shaped membership function as follows:

$$\mu(x) = \begin{cases} 1 & x < x^a \\ 0.999 & x = x^a \\ \dfrac{B}{1 + Ce^{\alpha x}} & x^a < x < x^b \\ 0.001 & x = x^b \\ 0 & x > x^b \end{cases} \quad (1)$$

where μ is the degree of membership function.

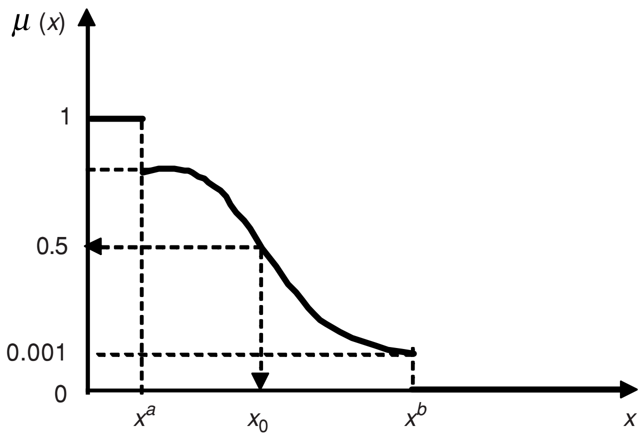

Figure 2 S-shaped Membership Function

Figure 2 shows the S-curve . In Equation 1 the membership function is redefined as $0.001 \leq \mu(x) \leq 0.999$. This range is selected because in manufacturing system the work force need not be always 100% of the requirement. At the same time the work force will not be 0%. Therefore there is a range between x^0 and x^1 with $0.001 \leq \mu(x) \leq 0.999$. This concept of range of $\mu(x)$ is used in this paper.

We rescale the x axis as $x^a = 0$ and $x^b = 1$ in order to find the values of B, C and α. Novakowska [10] has performed such a rescaling in his work of social sciences.

The values of B, C and α are obtained from Equation 1 as

$$B = 0.999(1 + C) \tag{2}$$

$$\frac{B}{1 + Ce^{\alpha}} = 0.001 \tag{3}$$

By substituting Equation 2 into Equation 3:

$$\frac{0.999(1 + C)}{1 + Ce^{\alpha}} = 0.001 \tag{4}$$

Rearranging Equation 4

$$\alpha = \ln \frac{1}{0.001}\left(\frac{0.998}{C} + 0.999\right) \tag{5}$$

Since, B and α depend on C, we require one more condition to get the values for B, C and α

Let, when $x_0 = \frac{x^a + x^b}{2}$, $\mu(x_0) = 0.5$; Therefore

$$\frac{B}{1 + Ce^{\frac{\alpha}{2}}} = 0.5 \tag{6}$$

and hence

$$\alpha = 2 \ln\left(\frac{2B - 1}{C}\right) \tag{7}$$

Substituting Equation 5 and Equation 6 in to Equation 7, we obtain

$$2 \ln\left(\frac{2(0.999)(1 + C) - 1}{C}\right) = \ln \frac{1}{0.001}\left(\frac{0.998}{C} + 0.999\right) \tag{8}$$

Rearranging Equation 8 yields

$$(0.998 + 1.998C)^2 = C(998 + 999C) \tag{9}$$

Solving Equation 9 :

$$C = \frac{-994.011992 \pm \sqrt{988059.8402 + 3964.127776}}{1990.015992} \tag{10}$$

Since has to be positive, Equation 10 gives $C = 0.001001001$ and from Equations 2 and 7, $B = 1$ and $\alpha = 13.81350956$.

S-shaped membership function is proved to be a flexible membership function through an analytical approach. This membership function is to be used in FLP involving fuzzy objective coefficients, fuzzy technical coefficients and fuzzy resource variables.

The objective of the company Chocoman Inc is to maximize its profit, which is, alternatively, equivalent to maximizing the gross contribution to the company in terms of US$. That is to find the optimal product mix under uncertain constraints in technical, raw material and market consideration. Furthermore, it is possible to show the relationshi p between the optimal profits and the corresponding membership grades [3]. According to this relationship, the decision maker can then obtain his optimal solution with a trade-off under a pre-determined allowable imprecision.

Through the use of fuzzy set theory and its interactive and problem-oriented concepts, the flexibility of IFLP techniques can be improved [11]. The IFLP approach can then be developed and this is an integration of various proposals [3,12,13,14]. This integration provides a decision support system for solving a specific domain of real-world IFLP systems.

5.0 FORMULATION OF FUZZY PRODUCT MIX - SELECTION PROBLEM

The Fuzzy Product – mix Selection Problem (FPSP) is stated as:

> *There are n products to be manufactured by mixing m raw materials with different proportion and by using k varieties of processing. There are limitations in resource of raw materials. There are also p constraints imposed by marketing department such as product – mix requirement, main product line requirement and lower and upper limit of demand for each product. All the above requirements and conditions are fuzzy. It is necessary to obtain maximum profit with certain degree of satisfaction by using interactive fuzzy linear programming.*

Chocoman Inc manufactures 8 varieties chocolate products. There are 8 raw materials to be mixed in different proportions and 9 processes (facilities) to be utilized.

Then, this FPSP is modeled by a FLP problem as

$$\begin{aligned} &Maximize\ z = c^T x \\ &Subject\ to\ Ax \leq b \end{aligned} \tag{11}$$

where c ia a row vector $(1xn)$ *objective coefficient,* $\tilde{A}$ *is matrix (mxn) a fuzzy technical coefficient and* $\tilde{b}$ *is a column vector* $(mx1)$ *fuzzy resource variable; z is the scalar objective function to be maximized inform of independent* $(nx1)$ *variables x.*

6.0 COMPUTATIONAL PROCEDURE

The FLP is solved by using MATLAB and its Linear programming tool box. Let the vagueness be given by a and m be the degree of satisfaction. This tool box has two inputs a and m in addition to the fuzzy intervals for objective coefficients, technical coefficients and resource variables. There is a single output z^*, the optimum profit.

The given values of varies parameters from Chocoman are fed to the tool box. The fuzzy data for objective coefficient, technical coefficient and resource variable are available [15].

7.0 COMPUTATION OF THE OBJECTIVE FUNCTION Z^*

The optimal solution for FPSP is obtained and it is shown in Table 1 and Figure 3.

From Figure 3, we can see that the graph behaves as an increasing function. This shows that the objective values increase as degree of satisfaction increases. The profit function (objective value) has a value 262,000 at $\mu = 1$. We define this as 100% degree of satisfaction. Accordingly, a z^* of value 252,770 has 0.1% degree of satisfaction. The possible realistic solution exists at $\mu = 0.5$ (ie 50% degree of satisfaction) with a value of z^* as 258,360. This value of z^* has to coincide with the realistic non fuzzy situation [7]. The non fuzzy situation (ie all the coefficients a_{ij}, c_j and b_i are precise) and the z^* value has been computed to be 254,400 [15]. It is found that z^* becoming more than that of a totally non fuzzy situation.

8.0 OBJECTIVE VALUES FOR VARIOUS A

Figure 3 illustrates the variation of objective values z^* with respect to degree of satisfaction m for one value of vagueness factor $a = 13.81350956$. It will be useful for the decision maker to observe such variations for several values of a. Figure 4, shows the nature of variations of z^* with respect to m when a varies from 2 to 20.

The membership value μ in Figure 4 represents the degree of satisfaction and z^* is profit function. We can conclude that when the vagueness increases, the profit value for a particular m decreases. This phenomenon actually happens in real life problems in a fuzzy environment.

Table 1 Optimal Solutions with S-curve Membership Function

Degree of Satisfaction (μ)	**Optimal Values ($z^{*} \times 10^{5}$)**
0.0010	2.5477
0.0509	2.5684
0.1008	2.5722
0.1507	2.5746
0.2006	2.5764
0.2505	2.5779
0.3004	2.5792
0.3503	2.5804
0.4002	2.5815
0.4501	2.5825
0.5000	2.5836
0.5499	2.5846
0.5998	2.5857
0.6497	2.5868
0.6996	2.5880
0.7495	2.5893
0.7994	2.5907
0.8493	2.5925
0.8992	2.5949
0.9491	2.5987
0.9990	2.6193

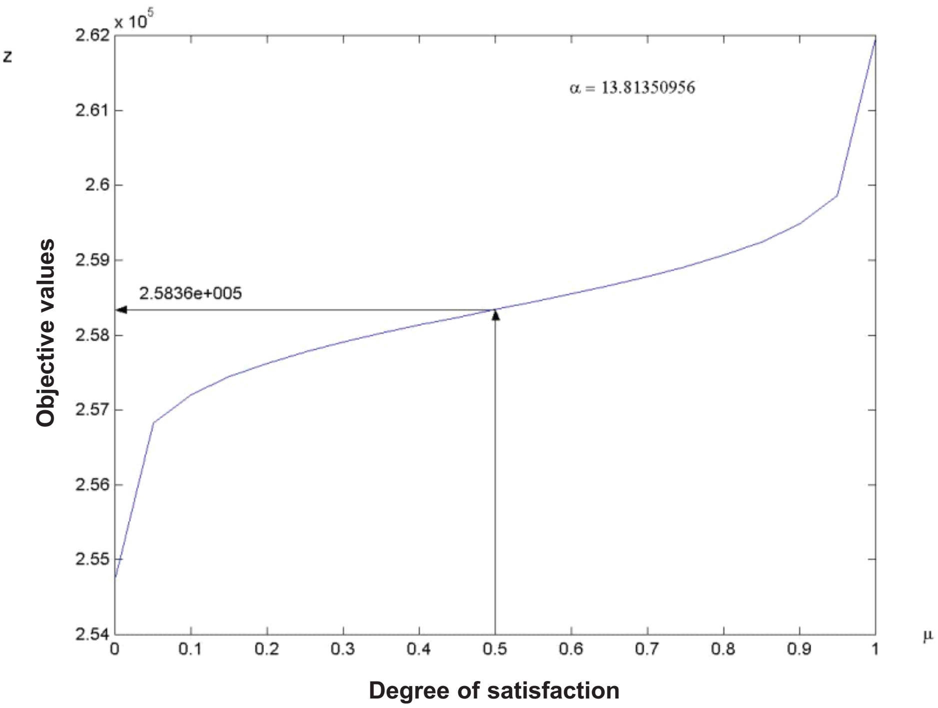

Figure 3 Degree of Satisfaction and Objective Values for $\alpha = 13.81350956$

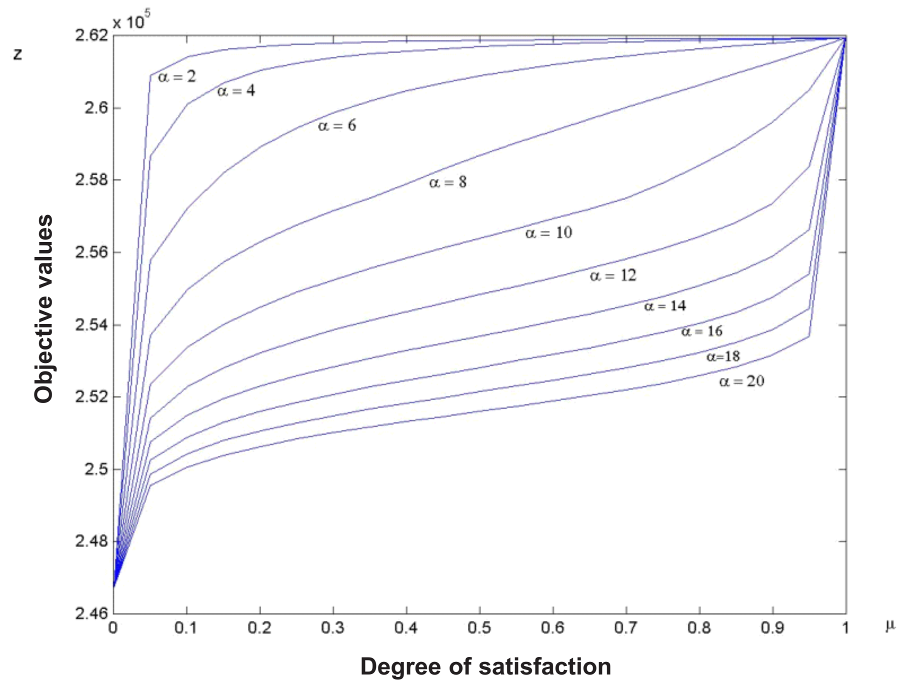

Figure 4 Degree of Satisfaction and Objective Values for $2 \leq \alpha \leq 20$

The ideal solution in a fuzzy environment exists at $\mu = 0.5$ [16]. Hence the result for 50% degree of satisfaction ($\mu = 0.5$) for $2 \leq \alpha \leq 20$ and the corresponding values for z^* are presented in Table 2.

Table 2 Fuzzy Parameter a, and Objective Values z^* (50% degree of satisfaction)

Fuzzy Parameter (Vagueness a)	Objective Value ($z^* \times 10^5$)
2	2.6191
4	2.6184
6	2.6153
8	2.6069
10	2.5968
12	2.5888
14	2.5829
16	2.5785
18	2.5751
20	2.5724

It can be seen from Table 2 that for $\mu = 0.5$ and as a increases, z^* decreases. It can be concluded that when the vagueness in the variables of objective coefficients increases, z^* decreases for same degree of satisfaction. The data in Table 2 is the result of analyzing the IFLP of Equation 11. These data are very useful for the decision maker to take a specific decision towards implementation after consulting the implementer.

The 3D plot of z^* for various value of vagueness α and degree of satisfaction μ is given in Figure 5.

It is found that the selected S shaped logistic membership function with various value of α offers an acceptable solution with certain degree of satisfaction in fuzzy environment. More vagueness results in less profit.

In Table 3, it can be seen that at $z^* = 259{,}000$ and vagueness $\alpha = 2$, degree of satisfaction is 2.0%. This shows that if the availability of material, labor and processing happen in less fuzzy environment (with $\alpha = 2$) then a profit of $259,000 will be obtained at 2.0% degree of satisfaction. Similarly when vagueness $\alpha = 18$ with highly fuzzy environment then the same profit of 259,000 is obtained at 95% degree of satisfaction. Finally it can be concluded that when vagueness increases, the degree of satisfaction also increases at any fixed objective value z^*.

The relationship between z^*, μ and α is given in the Table 4. This Table is very useful for the decision maker to find the profit value at any given value of μ with

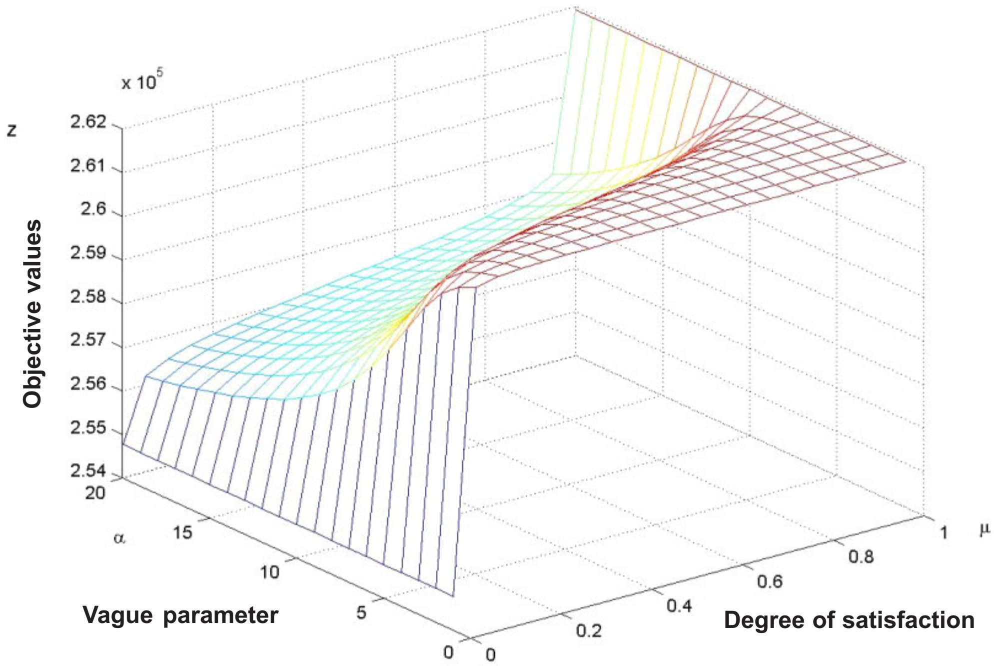

Figure 5 Degree of Satisfaction (μ) ,Vagueness (α) and Objective Values (z^*)

Table 3 Vagueness and Degree of satisfaction at $z^* = 2.59 \times 10^5$

Vagueness (α)	**Degree of Satisfaction (μ)**
2	2.00%
4	3.00%
6	4.50%
8	11.0%
10	26.5%
12	53.5%
14	79.0%
16	92.0%
18	95.0%
20	96.0%

degree of satisfaction μ. From the Table it can be seen that the objective value does not linearly dependent on the vagueness and the degree of satisfaction. One cannot conclude that for higher value of degree of satisfaction the profit value will be higher. This is not true. At 100% degree of satisfaction the profit value will be largest even with the higher value of vagueness. From the diagonal values in the Table, it can be concluded that the objective value increases at lower value of μ ($0.001 \le \mu \le 0.2505$). Then z^* value decreases for $0.5 \le \mu \le 0.7495$. Lastly z^* value increases for $0.7495 \le \mu \le 1$. This result shows that the good decision (higher degree of satisfaction) does not guaranty higher value in profit (objective value). This means one should satisfy with certain degree of satisfaction when come to making decision in a fuzzy environment.

Table 4 Distribution of z^* against μ and α

$z^* \times 10^5$ ↘	**Vagueness α** →				
↓ **Degree of Satisfaction (μ)**	**1**	**5**	**9**	**13**	**17**
0.0010	2.5477	2.5477	2.5477	2.5477	2.5477
0.2505	2.6189	2.6140	2.5935	2.5796	2.5721
0.5000	2.6192	2.6173	2.6017	2.5857	2.5767
0.7495	2.6193	2.6186	2.6089	2.5917	2.5813
0.9990	2.6193	2.6193	2.6193	2.6193	2.6193

9.0 CONCLUSION

This paper has illustrated the application of the proposed S-curve membershi p function in a real world industrial engineering problem, the chocolate manufacturing. The real life problem has 8 decision variables with 29 constraints to be included in the FLP formulation. This real life problem has been solved and their results are tabulated. The decision maker can also suggest to the analyst some possible and practicable changes in fuzzy intervals for improving the maximum profit. This interactive process has to go on among the analyst, the decision maker and the implementer till an optimum satisfactory decision is achieved and implemented.

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