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Computed Torque and Velocity Feedback Control of Cooperative Manipulators Handling a Flexible Beam

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ABSTRACT

Handling a flexible object is more complicated than a rigid one since it involves the vibration of the object. Since vibration is known to lead to disturbance, discomfort, damage, and destruction; it needs to be suppressed. The system consists of two cooperative manipulators handling a flexible beam that is modelled in partial differential equation (PDE) form and employed a singular perturbation method to model the slow and fast subsystems. This paper presents a composite control comprising of the computed torque control (CTC) scheme for the slow subsystem and a velocity feedback control (VFC) for the fast subsystem that was developed based on the PDE model form so that two cooperative manipulators track the desired trajectories while suppressing the transverse vibrations of the beam. A stability analysis was carried out for each subsystem to satisfy Tikhonov's Theorem. The simulation results for slow subsystem showed that the tracking of positions and orientation have been achieved within 0.5 s with the root-mean-square error (RMSE) values of 0.002745 m, 0.02292 m, and 0.01563 rad for X-direction, Y-direction and the orientation, respectively. For the fast subsystem, the transverse vibration of the beam is completely suppressed within 0.8 s. The results proved that the proposed controller has worked well with the PDE model of cooperative manipulators to handle the flexible beam while suppressing its vibration.

Keywords: Cooperative Manipulators, Flexible Beam, Singular Perturbation Method, Computed Torque Control, Slow-Fast Subsystems.

1. INTRODUCTION

There are several applications of a manipulator system in industries such as the transportation of massive objects in the manufacturing industry, assembling parts in the automotive industry, and handling objects in the food industry. The manipulator robot does not only need to handle rigid objects, but also flexible or deformable ones such as sheet metals, rubber tubes, cords and leather products. Handling flexible objects becomes more challenging, especially when they are long, delicate or/and heavy. It requires at least two manipulators that work cooperatively so that they can be handled safely and efficiently [1]. Flexible objects are objects that always have relatively low rigidity and small structural damping. Large-amplitude, long decay time and an infinite number of the mode of vibrations are produced when there is an excitation acting on it. Thus, this vibration needs to be suppressed since it can lead to disturbance, discomfort, damage and destruction. Several studies have reported the handling of three-dimensional deformable objects. General demonstrations-based learning manipulation of deformable objects by two cooperative robots was studied such as tying and tightening a knot, tying a rope to a pipe, flattening and folding a towel, and erasing a whiteboard [2]. An automatic three-dimensional manipulation of soft objects by robotic arms was also discussed [3]. The model-free approach was addressed to control robots with unknown dynamics and manipulating flexible rubber objects of unknown elasticity [4].

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A different approach on handling a deformable object was presented by designing a novel electric gripper with parallel fingers in which each finger has a set of an independent drive motor and integrated with a force sensor [5].

Modelling a flexible object is challenging as it has an infinite number of degree-of-freedom (DOF). In previous works, it was modelled as a simple spring system [6], spring-damping system [7] and the approximated model of a flexible beam. The Finite Element Method (FEM) [8][9] and Assume Mode Method (AMM) [10]–[14] are amongst the popular methods in approximating the model of the flexible beam to be incorporated with the control designs. However, these methods have truncated the original model with an infinite number of DOF of a flexible beam to a finite-dimensional model that leads to several drawbacks in which it can be improved by describing the dynamics of the flexible beam in PDE form [15][16]. The mentioned PDE model has several advantages: it can avoid the use of many sensors to measure the vibration, it considers the higher-order modes that can lead to system destabilization, and it prevents control and observation spill-over [16]. Due to the advantages offered by PDE system, the control strategy based on PDE-based model has been used in many current studies regarding the various cases of flexibility control such as the flexible-link manipulator [17]–[22], flexible plate [23], flexible beam [24] and flexible inverted pendulum system [25].

The control of cooperative manipulators is more complicated than the control of a single manipulator. A synchronization motion of two cooperative manipulators is restricted by a flexible beam that can cause vibration. Hence, a boundary control (BC) law was proposed to be incorporated with a position control law to realize the synchronization motion of two manipulators and the regulation of position errors [26]. Both model-based control and adaptive barrier control were developed for the beam system with boundary output constraint to suppress the vibration of the beam without violation of the constraint [27]. A model-based method was proposed to calculate the inverse and direct dynamic model of cooperative manipulators in handling flexible objects [28]. Force control of dual-manipulator handling of a flexible payload based on the distributed parameter model was introduced to achieve the desired contact force and desired link angles as well as to suppress the vibrations of the flexible payload [29]. A different approach without employing any force/torque measurements which is a novel control protocol was proposed for the cooperative manipulation of an object by N robotic agents using unit quaternions [30].

The two-time scales (TTS) control is a well-known approach comprising of slow control to track the desired trajectories and fast control to control the vibration of the flexible beam that is designed for slow and fast subsystems, respectively. For the in-plane motion of the flexible beam, the inverse dynamics control for slow subsystem was studied so that the centre of mass of the flexible beam follows the desired positions/orientation and the asymptotic stabilizing BC law for the fast subsystem was designed to suppress the beam's vibration [31]. Then, the stability of the fast subsystem was also proven by using exponential stabilising BC law [32] in which it is more successful than the asymptotic stabilising BC law in removing undesirable vibrations of the flexible beam. This idea has been extended to two cooperative manipulators handling a flexible beam by proposing sophisticated controllers for slow subsystem such as the regressor-based robust sliding mode controller [16], the robust adaptive controller [15], and velocity feedback controller (VFC) for a fast subsystem. However, the aforementioned PDE-based model of two cooperative manipulators handling the flexible beam has yet to be tested by implementing a simpler controller such as computed torque control (CTC).

Therefore, this paper presents the implementation of the CTC scheme for a slow subsystem and velocity feedback control (VFC) for a fast subsystem for two cooperative manipulators handling the flexible beam that is modelled based on PDE. CTC is a simple and easy controller that can be used as a starting point to validate the PDE-based model of two cooperative manipulators handling the flexible beam. CTC has several advantages; for example, it can linearise the nonlinear dynamics of the system, provide excellent tracking performance, circumvent the problems of uncertainties by utilizing adaptive techniques and often work well in practice [33]–[35]. The structures of the paper are organized as follows: Section 2 provides the kinematics of the cooperative manipulators and the flexible beam. Section 3 describes the dynamics of the cooperative manipulators and the flexible beam, and the combined dynamics that employ the singular perturbation method to produce slow and fast subsystem models. Section 4 presents the design of the composite control consists of CTC scheme for the slow subsystem and VFC for fast subsystems. The stability analysis of the composite control must satisfy Tikhonov's theorem. The simulation results are presented in Section 5 to prove the feasibility of the proposed controllers. Finally, Section 6 concludes the paper.

2. MATERIAL AND METHODS

The system consists of two identical and planar cooperative manipulators that handle a flexible beam. Each manipulator has three rigid links with three revolute joints. The schematic representation of manipulators with corresponding joint angles, q_{ij} and link lengths, l_{ij} where $i = 1,2$ represents i-th manipulator and represents j-th link. For each manipulator, the first end of the first link is known as the base, the second end of the second link is the wrist, and the end-effector is attached to the second end of the third link as shown in Figure 1.

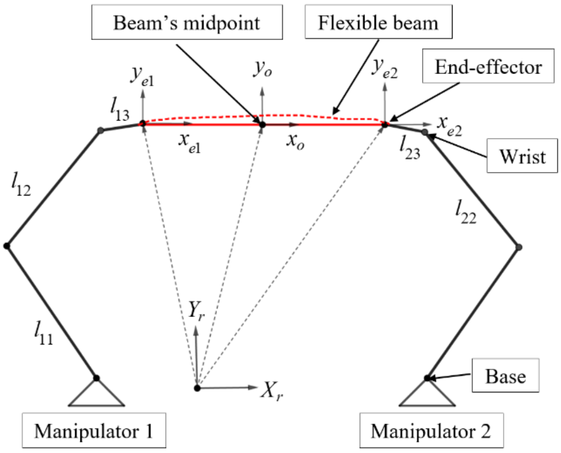

Figure 1. Two planar and cooperative manipulators handling a flexible beam.

2.1 Kinematics of the Cooperative Manipulators

The forward kinematics of planar manipulators is the relationship between the manipulators' joint angles in polar coordinate and its end-effectors in Cartesian space. It is needed for modelling the system and designing a controller such as CTC. Since two manipulators are assumed to be identical, the forward and inverse kinematics are applicable for both manipulators. For i-th manipulator, where $i = 1,2$, the forward kinematics is described as

$$\begin{aligned} x_{ei} &= x_{bi} + l_{i1}\cos(q_{i1}) + l_{i2}\cos(q_{i1}+q_{i2}) + l_{i3}\cos(q_{i1}+q_{i2}+q_{i3}) \\ y_{ei} &= y_{bi} + l_{i1}\sin(q_{i1}) + l_{i2}\sin(q_{i1}+q_{i2}) + l_{i3}\sin(q_{i1}+q_{i2}+q_{i3}) \end{aligned} \tag{1}$$

where x_{ei} and y_{ei} are x-position and y-position of the end-effectors of i-th manipulator, respectively; meanwhile, x_{bi} and y_{bi} are x-position and y-position of the base of i-th manipulator, respectively.

It is noted that all the positions are with respect to X_rY_r -frame, as shown in Figure 2.

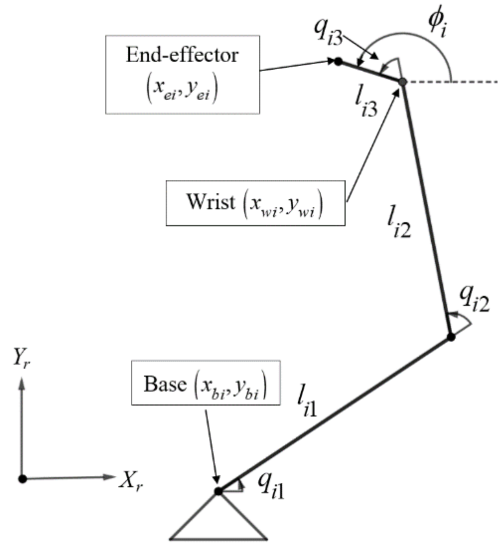

Figure 2. Three degree of freedom (DOF) planar of the *j*th manipulator with three revolute joints.

2.2 Kinematics of the Flexible Beam

The length, L and mass, $m = \rho L$ of a flexible beam, is considered where ρ is mass per unit length. The coordinate frames of X_rY_r is fixed frame and $xy -$ frame is a moving coordinate frame which is attached to the beam's midpoint, X_{mp} [16] given by

$$X_{mp} = \{x_o \quad y_o \quad \theta\}^T, \tag{2}$$

where $x_o, y_o,$ and θ represent x-position, y-position and the orientation of the beam's midpoint, respectively. Meanwhile, $F_{1x}, F_{1y}, F_{2x},$ and F_{2y} are the forces applied by the manipulators at the two ends of the beam, as shown in Figure 3.

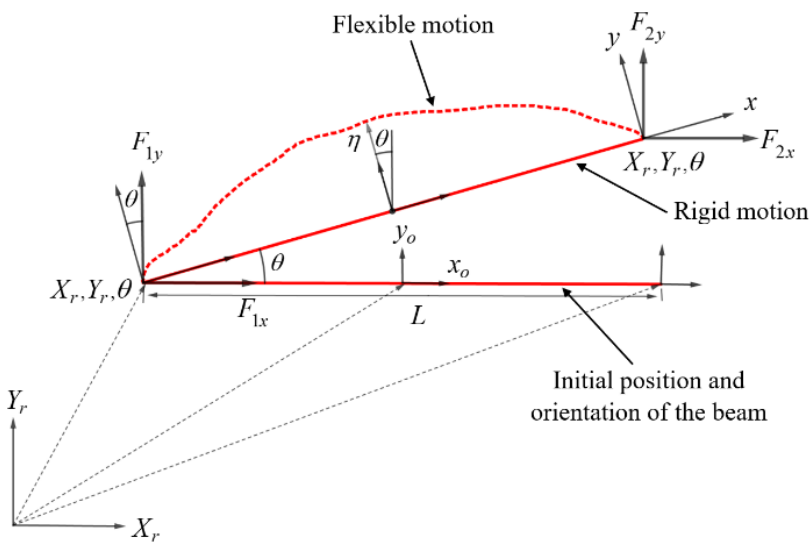

Figure 3. Schematic diagram of the flexible beam.

The transverse displacement, $\eta(x,t)$ is measured with respect to xy -frame in which it varies with time, t and spatial coordinate, x that ranges from $-L/2$ to $+L/2$ [16]. The argument (x,t) will be omitted in the next section of this paper for simplicity. The kinematics analysis and trajectory validation of two cooperative manipulators handling a flexible beam are described further in [36].

2.3 Dynamics of the System

The dynamics of the system comprises cooperative manipulators dynamics and flexible beam dynamics that are combined to form a combined dynamic. Then, the singular perturbation method was employed to yield a singular perturbation model [37].

2.3.1 Dynamics of Cooperative Manipulators

The general dynamics equation for 3-DOF and planar manipulator [38] can be expressed as

$$M_i(q_i)\ddot{q}_i + C_i(q_i,\dot{q}_i)\dot{q}_i + G_i(q_i) = \tau_i + J_i^T f_i \, , \tag{3}$$

Where

$$\begin{gathered} M_i(q_i) = \begin{bmatrix} m_{i11} & m_{i12} & m_{i13} \\ m_{i21} & m_{i22} & m_{i23} \\ m_{i31} & m_{i32} & m_{i33} \end{bmatrix}, C_i(q_i,\dot{q}_i) = \begin{bmatrix} c_{i11} & c_{i12} & c_{i13} \\ c_{i21} & c_{i22} & c_{i23} \\ c_{i31} & c_{i32} & c_{i33} \end{bmatrix}, \\ G_i(q_i) = \begin{Bmatrix} g_{i1} \\ g_{i2} \\ g_{i3} \end{Bmatrix}, J_i(q_i) = \begin{bmatrix} J_{i11} & J_{i12} & J_{i13} \\ J_{i21} & J_{i22} & J_{i23} \\ 1 & 1 & 1 \end{bmatrix}, \\ \tau_i = \begin{Bmatrix} \tau_{i1} \\ \tau_{i2} \\ \tau_{i3} \end{Bmatrix}, f_i = \begin{Bmatrix} f_{i1} \\ f_{i2} \\ f_{i3} \end{Bmatrix}, \dot{q}_i = \begin{Bmatrix} \dot{q}_{i1} \\ \dot{q}_{i2} \\ \dot{q}_{i3} \end{Bmatrix}, \ddot{q}_i = \begin{Bmatrix} \ddot{q}_{i1} \\ \ddot{q}_{i2} \\ \ddot{q}_{i3} \end{Bmatrix} \end{gathered} \tag{4}$$

and $M_i(q_i)$represents 3×3 symmetric positive definite inertia matrix, $C_i(q_i,\dot{q}_i)$is 3×3 Coriolis and Centrifugal matrix, $G_i(q_i)$ is 3×1 gravitational components vector, $J_i(q_i)$ is 3×1 manipulators the Jacobian matrix, τ_i is 3×1 vectors of input torque applied at each joint of the manipulators, f_i is 3×1vector of interaction force between the manipulators and the flexible beam, and $q_i, \dot{q}_i$ and$\ddot{q}_i$ represent 3×1 vectors of generalised joint displacements, velocities and accelerations, respectively.

All elements in each matrix or vector [38] in equation (4) are presented in Appendix A. For two cooperative manipulators, the dynamics equations can be written in joint space as

$$M_m\ddot{q} + C_m\dot{q} + G_m = \tau + J^T f \, , \tag{5}$$

Where

$$\begin{gathered} M_m = \begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix}; C_m = \begin{bmatrix} C_1 & 0 \\ 0 & C_2 \end{bmatrix}; J = \begin{bmatrix} J_1 & 0 \\ 0 & J_2 \end{bmatrix}; G_m = \begin{Bmatrix} G_1 \\ G_2 \end{Bmatrix}; \\ \dot{q} = \{\dot{q}_1 \quad \dot{q}_2\}^T ; \ddot{q} = \{\ddot{q}_1 \quad \ddot{q}_2\}^T ; \tau = \{\tau_1 \quad \tau_2\}^T ; f = \{f_1 \quad f_2\}^T \end{gathered} \tag{6}$$

It is noted that $M_i, C_i, G_i, J_i, \tau_i, f_i, \dot{q}_i$, and $\ddot{q}_i$ are from equations (3) and (4), where $i = 1{,}2$ that represents i-th manipulator.

2.3.2 Dynamics of the Flexible Beam

The dynamics of the beam is derived by using the extended Hamiltonian Principle as:

$$\int_{t_1}^{t_2} \left(\delta U - \delta T - \delta W\right) dt = 0 \tag{7}$$

where δU, δT and δW are a variation of potential energy, U, kinetic energy, T and work done due to external forces, W respectively. Meanwhile, t_1 and t_2 are any two instances of time with $t_2 > t_1 > 0$.

The dynamics of the beam is presented in Cartesian coordinates comprising of rigid dynamics and transverse vibration, which is flexible dynamics. The rigid dynamics of the beam which is written in the compact form [16] is presented as:

$$M_{brf}\ddot{X}_{mp} + C_{brf} + \eta_{brf} + G_{brf} = F_{brf}(-f), \tag{8}$$

where

$$M_{brf} = \begin{bmatrix} m & 0 & M_{brf1} \\ 0 & m & M_{brf2} \\ M_{brf1} & M_{brf2} & M_{brf3} \end{bmatrix}, C_{brf} = \begin{Bmatrix} C_{brf1} \\ C_{brf2} \\ 0 \end{Bmatrix}, \eta_{brf} = \begin{Bmatrix} \eta_{brf1} \\ \eta_{brf2} \\ \eta_{brf3} \end{Bmatrix},$$

$$G_{brf} = \begin{Bmatrix} 0 \\ mg \\ 0 \end{Bmatrix}, \ddot{X}_{mp} = \begin{Bmatrix} \ddot{x}_0 \\ \ddot{y}_0 \\ \ddot{\theta} \end{Bmatrix}, F_{brf} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ F_{brf1} & F_{brf2} & 0 & F_{brf3} & F_{brf4} & 0 \end{bmatrix},$$

$$f = \begin{Bmatrix} F_{1x} & F_{1y} & M_{O_1} & F_{2x} & F_{2y} & M_{O_2} \end{Bmatrix}^T$$

Since the manipulators have 3-DOF, $M_{brf}, C_{brf}, \eta_{brf}, G_{brf}, F_{brf}, f$, and $\ddot{X}_{mp}$ represent 3×3 inertia matrix of the beam, 3×1 centrifugal vector of the beam, 3×1 vibration vector of the beam, 3×1 gravitational vector of the beam, 3×6 force transformation matrix, 6×1 forces/moments vector at the two ends of the manipulator, and 3×1 acceleration vector of the beam's midpoint, respectively. The subscript "*brf*" is used to denote a matrix or vector that consists of flexible and rigid parameters. All elements in each matrix or vector [16] in Equation (8) are presented in Appendix B. The transverse vibration of the beam is presented as:

$$-\sin\theta\ddot{x}_o + \cos\theta\ddot{y}_o + x\ddot{\theta} + \ddot{\eta} - \eta\dot{\theta}^2 + \frac{EI}{\rho}\eta^{iv} = F_{ff}\left(f\right), \tag{9}$$

where

$$F_{ff} = \frac{1}{m}\left[-\sin\theta \quad \cos\theta \quad 0 \quad -\sin\theta \quad \cos\theta \quad 0\right] \tag{10}$$

and $F_{ff}, f, \eta, \ddot{\eta}, \eta^{iv}, E$, and I are 1×6 force transformation matrix in the transverse vibration, 6×1 forces/moments vector at the two ends of the 3-DOF manipulator pair, transverse displacement, $\eta(x,t)$ that varies with x and t, second derivative of $\eta(x,t)$ with respect to t, fourth derivative of $\eta(x,t)$ with respect to x, moment of inertia of the beam (kgm^2), and Young's modulus of the beam (Pa).

2.3.3 Combined Dynamics and Singular Perturbation Model

The rigid dynamics of both cooperative manipulators (5) and the beam (8) are combined to form combined rigid dynamics while the transverse vibration is still the same as (9) [16] in which the rigid and flexible parameters are involved in the equations. Due to highly non-linear systems, the singular perturbation technique is employed on the combined rigid dynamics and transverse vibration to approximate the solutions and reduce the order of the model of the system. As a result, two subsystems of the singular perturbation model with different time scales, namely the slow subsystem and fast subsystem, are produced. This is the basic idea of the two-time scales theory [37]. The slow subsystem is produced when the flexible parameter in combined rigid dynamics and transverse vibration are decoupled and eliminated by introducing a new variable, $v(x,t)$ in the same order of the state variable [16] as:

$$v(x,t) = \mu^2 . \eta(x,t), \tag{11}$$

where $\mu = 1/C$ is known as the perturbed parameter and C is a dimensionless parameter which has a large value for different materials.

The flexible parameter is eliminated by setting the perturbed parameter, μ approaching to zero ($\mu \to 0$). Therefore, the slow subsystem representing rigid body motion without involving any flexible parameters [16] is given as:

$$M_{cr} \ddot{X}_{mp} + C_{cr} \dot{X}_{mp} + G_{cr} = U_{cr}, \tag{12}$$

where

$$\begin{aligned} M_{cr} &= R_r^T J^{-T} M_m J^{-1} R_r + M_{br}; \quad G_{cr} = R_r^T J^{-T} G_m + G_{br} \\ C_{cr} &= R_r^T J^{-T} (M_m \dot{J}^{-1} R_r + M_m J^{-1} \dot{R}_r + C_m J^{-1} R_r); \quad U_{cr} = R_r^T J^{-T} \tau \end{aligned} \tag{13}$$

The fast subsystem is obtained by ensuring that the slow variable is kept constant in the fast time scale, h=$(t - t_0)/\mu$, where t, t_0 and μ is the slow time scale, slow initial time and perturbed parameter, respectively. Two variables which are slow variable, v_s and fast variable, v_f are defined and related as $v_f = v - v_s$. After going through some derivations, the fast subsystem that represents transverse vibration [16] yields:

$$\hat{\hat{v}}_f(h) + A\hat{v}_f = F_{ff}\left(f_f\right), \tag{14}$$

where the initial conditions are as follows: $v_f(0) = v_{f_o}, \dot{v}_f(0) = v_{f_1}$ and $\hat{v}_f, \hat{\hat{v}}_f, F_{ff}, A,$ and f_f represent the first derivative of fast variable, v_f with respect to fast time scale, h, second derivative of fast variable, v_f with respect to fast time scale, h, 1×6 force transformation matrix in the transverse vibration for the pair of 3-DOF manipulators, a differential operator in Hilbert space, and interaction force between a manipulator and the flexible beam in the fast subsystem, respectively.

This mathematical model of the fast subsystem will be used in designing the control algorithm to damp out the beam's vibration.

2.4 Composite Control Design

A controller is designed for each subsystem of the singular perturbation model to form a composite controller, U_c as:

$$U_c = U_s\left(\dot{X}_{mp}, X_{mp}, t\right) + U_f\left(\hat{v}_f, v_f, h\right), \tag{15}$$

where U_s is designed based on the slow subsystem (12) for trajectory tracking and U_f is designed to suppress the vibration in a fast subsystem model (14).

The composite control must satisfy Tikhonov's theorem, which states that if slow and fast dynamics are stable, then the stability of the combined dynamics is proven. Instead, if one of them is unstable, the whole system is unstable [39]. The block diagram of the control system is shown in Figure 4.

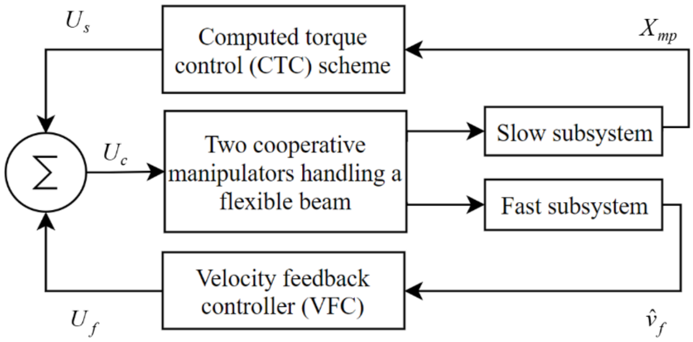

Figure 4. Block diagram of the control system.

2.4.1 Computed Torque Control (CTC) Scheme for the Slow Subsystem

The CTC scheme is designed for the slow subsystem to drive two cooperative manipulators in handling a flexible beam so that the positions and orientation of the centre of the beam, X_{mp} tracks its desired positions and orientation, X_{mpd} . Designing the scheme involves two parts which are the derivation of Inner Feedforward Loop and designing of Proportional-Plus-Derivative (PD) Feedback Outer Loop, as shown in Figure 5.

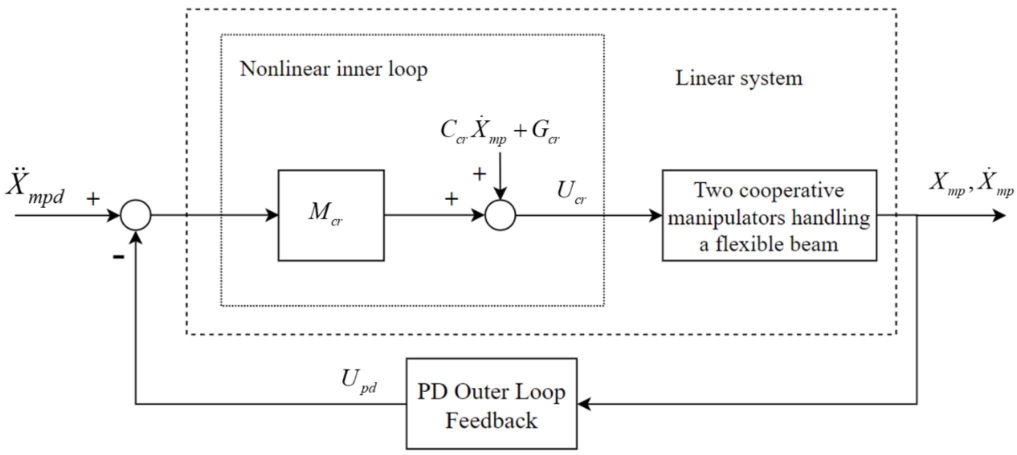

Figure 5. Computed torque control (CTC) scheme for slow subsystem.

2.4.2 Inner Feed-Forward Loop

Supposing that the desired trajectory, X_{mpd} has been selected for the motion of cooperative manipulators in handling a flexible beam, to ensure the trajectory tracking by the state variables, X_{mp}, the tracking error, $e(t)$ is defined as:

$$e(t) = X_{mpd} - X_{mp} \tag{16}$$

Differentiating (16) twice gives:

$$\ddot{e} = \ddot{X}_{mpd} - \ddot{X}_{mp} \tag{17}$$

Solving for $\ddot{X}_{mp}$ in (12) and substituting into (17) gives:

$$\ddot{e} = \ddot{X}_{mpd} + M_{cr}^{-1}\left[C_{cr}\dot{X}_{mp} + G_{cr} - U_{cr}\right] \tag{18}$$

Defining control input function, U_{pd} as:

$$U_{pd} = \ddot{X}_{mpd} + M_{cr}^{-1}\left[C_{cr}\dot{X}_{mp} + G_{cr} - U_{cr}\right] \tag{19}$$

and a state $x(t) \in R^{2n}$, where n is the number of DOF (in this case is 3-DOF) as:

$$x = \begin{bmatrix} e & \dot{e} \end{bmatrix}^T \tag{20}$$

and the tracking error dynamics can be written as:

$$\frac{d}{dt}\begin{bmatrix} e \\ \dot{e} \end{bmatrix} = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix}\begin{bmatrix} e \\ \dot{e} \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} U_{pd} \tag{21}$$

Inverting the feedback linearising transformation (19) yields the computed-torque control law [33] as:

$$U_s \equiv U_{cr} = M_{cr}\left[\ddot{X}_{mpd} - U_{pd}\right] + C_{cr}\dot{X}_{mp} + G_{cr} \tag{22}$$

These manipulations are significant because no state-space transformation has been involved from (12) to (21). Therefore, if the control input, $U_{pd}(t)$ in (19) is selected to stabilise the tracking error dynamics (21), so that $e(t)$ goes to zero, then the nonlinear control input, $U_{cr}(t)$ in (22) will cause the cooperative manipulators (12) to track the desired trajectories. Substituting (22) into (12) yields:

$$M_{cr}\ddot{X}_{mp} + C_{cr}\dot{X}_{mp} + G_{cr} = M_{cr}\left[\ddot{X}_{mpd} - U_{pd}\right] + C_{cr}\dot{X}_{mp} + G_{cr} \tag{23}$$

or

$$U_{pd} = \ddot{X}_{mpd} - \ddot{X}_{mp} \equiv \ddot{e} \tag{24}$$

which is exactly as (21).

2.4.3 PD Feedback Outer Loop

The PD feedback is selected to be the auxiliary signal, $U_{pd}(t)$ [33] as:

$$U_{pd} = -K_v \dot{e} - K_p e \tag{25}$$

In substituting (25) into (22), the overall cooperative manipulators' input, U_{cr} in (22) becomes:

$$U_s \equiv U_{cr} = M_{cr}\left[\ddot{X}_{mpd} + K_v \dot{e} + K_p e\right] + C_{cr}\dot{X}_{mp} + G_{cr} \tag{26}$$

The closed-loop error dynamics are:

$$\ddot{e} + K_v \dot{e} + K_p e = 0 \tag{27}$$

or in state-space form,

$$\frac{d}{dt}\begin{bmatrix} e \\ \dot{e} \end{bmatrix} = \begin{bmatrix} 0 & I \\ -K_p & -K_v \end{bmatrix}\begin{bmatrix} e \\ \dot{e} \end{bmatrix} \tag{28}$$

The closed-loop characteristics polynomial, $\Delta_c(s)$ is:

$$\Delta_c(s) = | s^2 I + K_v s + K_p | \tag{29}$$

The 3×3 diagonal matrices of PD gains [33] for the two cooperative manipulators with 3-DOF is:

$$K_v = diag\left\{k_{v_i}\right\},\ K_p = diag\left\{k_{p_i}\right\}, \tag{30}$$

Then

$$\Delta_c(s) = \Pi_{i=1}^{n}\left(s^2 + k_{vi}s + k_{p_i}\right) \tag{31}$$

and the error system is asymptotically stable if k_{v_i} and k_{p_i} are all positive.

2.5 Velocity Feedback Control (VFC) for Fast Subsystem

VFC is used because of its simple implementation in a real-time, limited number of sensors requirement and irrespective boundary conditions [16]. The objective of the controller is to suppress the vibration of the flexible beam by utilising the following velocity feedback control law

$$U_f = \left(f_f\right) = -\Pi F_{ff}^{\dagger}\hat{v}_f(h) \tag{32}$$

where $F_{ff}^{\dagger}$ is pseudo-inverse of F_{ff} [16]. The operator $\Pi = kQA$ is neither self-adjoint nor positive definite, where k is the positive gain, Q is a bounded and positive definite operator, and A is a positive definite operator [40], [41]. The stability analysis of the fast control can be found further in [16].

3. RESULTS AND DISCUSSION

The proposed approach has been simulated on two identical planar manipulators used to move a flexible beam while suppressing the beam's vibration. Each manipulator has three rigid links with three revolute joints. The parameters of the identical manipulators and the flexible beam have been given in Table 1 and Table 2, respectively. An open-loop model without any controller is simulated by assigning the desired values of system input, $U_{crd} = \{184.7\ N, 41.7\ N, 23.8\ N\}^T$ to test the constructed mathematical model in Simulink that consists of a slow subsystem (12) and fast subsystem (14). The results show that no programming errors have been produced. For the slow subsystem, it can be observed that there are random motions of the beam's midpoint, $X_{mp} = \{x_o, y_o, \theta\}$ which rapidly occur for the first 1.5 s. The ranges of the motions are in between -0.16 m to 4.55e-3 m for x-position, x_o, -0.20 m to 4.05e-2 m for y-position, y_o and -0.71 rad to 0.21 rad for the orientation, θ as shown in Figure 6. These random motions need to be controlled by the CTC scheme. For the fast subsystem, the vibration with the amplitude of 5 ms is induced, as shown in Figure 7. This vibration needs to be suppressed by the VFC law.

Table 1 Parameter of each manipulator

Link	Length (m)	Mass (kg)	Moment of inertia (kg)
1	0.29	1.50	1.06e-2
2	0.27	1.04	9.38e-3
3	0.13	0.13	8.15e-5

Table 2 Parameters of the flexible beam

Parameter	Value
Mass *(m)*	0.68 *kg*
Length *(L)*	0.45 *m*
Density *(p)*	2700 kgm^{-3}
Young's modulus *(E)*	71 *GPa*
Moment of inertia *(I)*	0.13 kgm^2

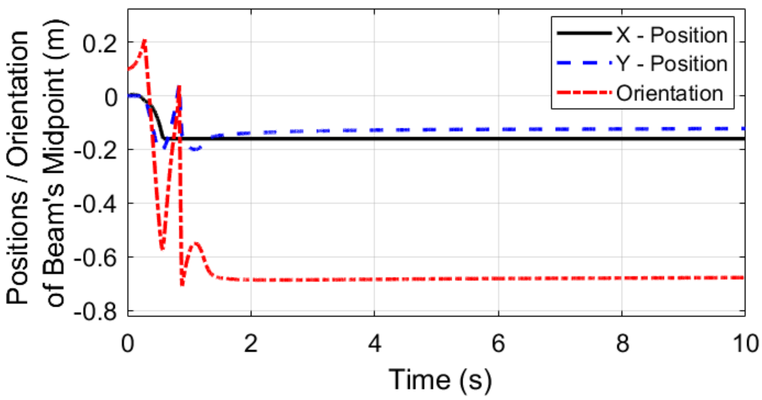

Figure 6. Positions/orientation of the beam's midpoint, X_{mp}.

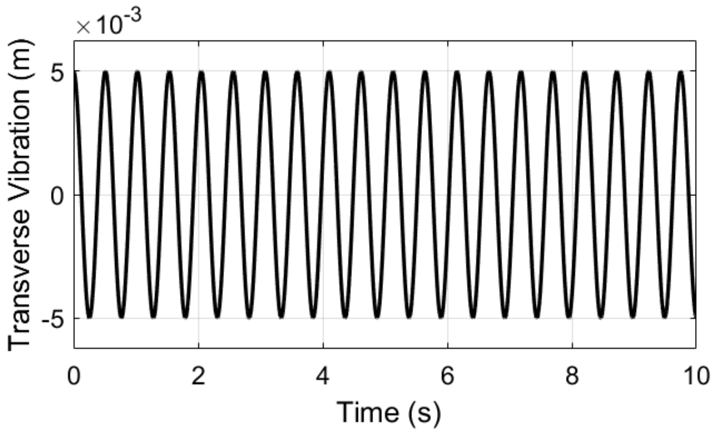

Figure 7. Vibration of the flexible beam, v_f.

For the closed-loop control system simulations, the slow (12) and fast (14) subsystems model are incorporated with CTC scheme (26) and VFC law (32), respectively. For the slow subsystem, the manipulators are commanded to move the flexible beam so that its midpoint positions and orientation, X_{mp} track the desired trajectories as [36]:

$$X_{mpd} = \left[0.1\sin(t) \quad 0.1\cos(t)+0.1 \quad 0\right]^T, \tag{33}$$

The initial positions and the orientation of the beam's midpoint are $X_{mp} = \{0\ m,\ 0\ m,\ 0.1\ rad\}^T$ while the initial velocity, $\dot{X}_{mp}$ and acceleration, $\ddot{X}_{mp}$ are specified as zero. The distance of the base of the two manipulators is considered as 1 m apart. The simulations are carried out by using ode2 typed solver with a sampling period of 0.001 s and simulation time of 10 s in MATLAB Simulink, as shown in Figure 8. The gains K_p and K_v are selected as diag{900 900 900} and diag{60 60 60}, respectively.

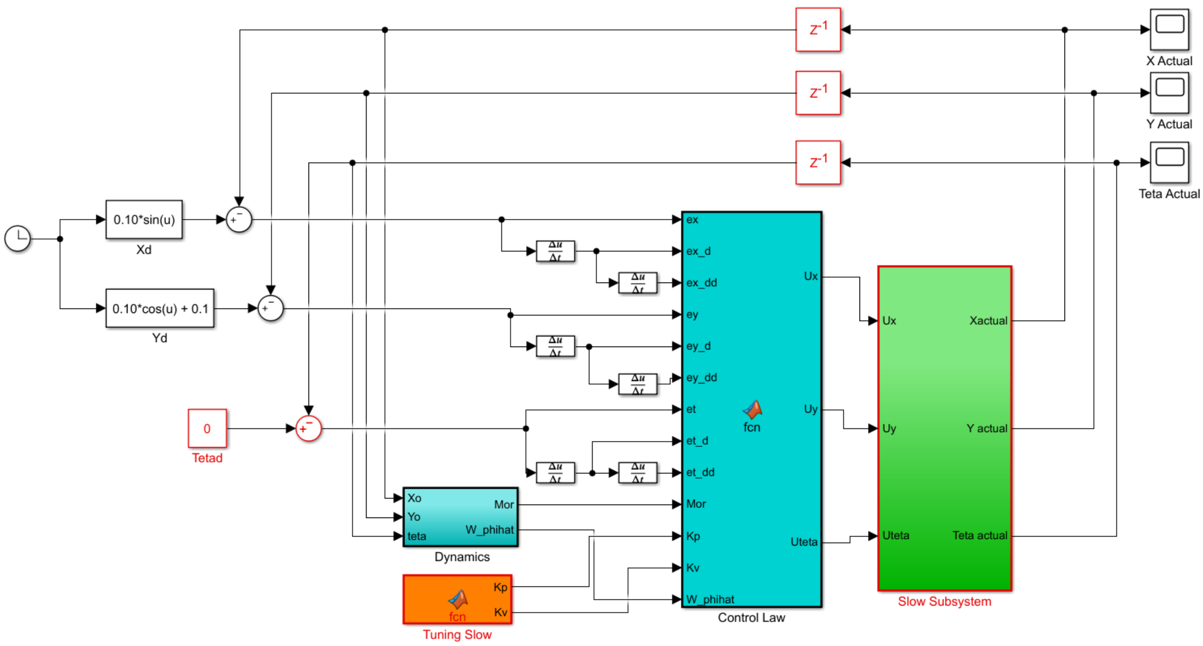

Figure 8. CTC scheme for the slow subsystem in Simulink.

The tracking of planar motions of the beam's midpoint along x, y-direction, and the beam's orientation are shown in Figures 9 to 11, respectively. The tracking errors of the beam's midpoint, X_{mp} are shown in Figure 12. It can be observed that the tracking of positions and orientation are achieved within 0.5 s with the root-mean-square (RMS) error values of 0.002745 m, 0.02292 m, 0.01563 rad. for x, y-directions and the orientation, respectively. These values clearly show the feasibility and effectiveness of the proposed control scheme on the PDE-based model of two cooperative manipulators handling the flexible beam. In the case of the fast subsystem, the simulation considered the initial disturbance of 5 mm with zero initial velocity. The control parameters of λ and k are chosen as diag{80} and 1, respectively. Similar to the slow subsystem, the simulations are carried out by using ode2 typed solver with a sampling period of 0.001 s and simulation time of 10 s in MATLAB Simulink, as shown Figure 13. With the value of $\beta = -1/2$, it can be observed that the transverse vibration of the beam is completely suppressed at around 0.8 s as shown in Figure 14. The results proved that the CTC scheme has successfully driven the PDE-based model of cooperative manipulators handling the flexible beam to follow the desired trajectory accurately and reduce the beam's vibration.

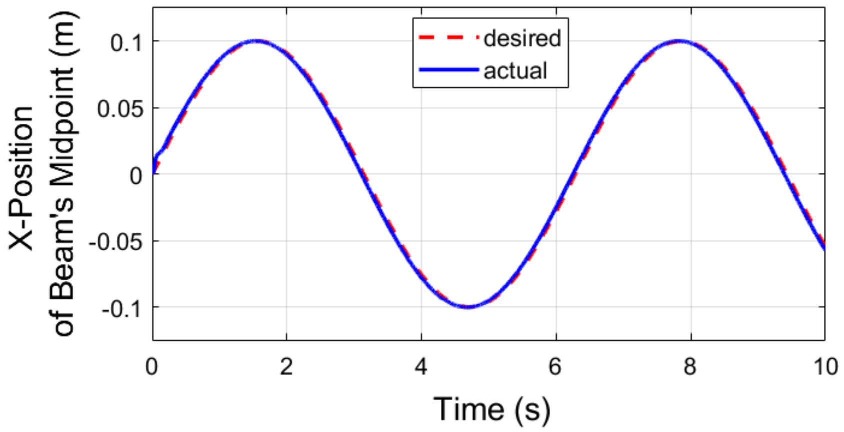

Figure 9. X-position tracking of the beam, x_o.

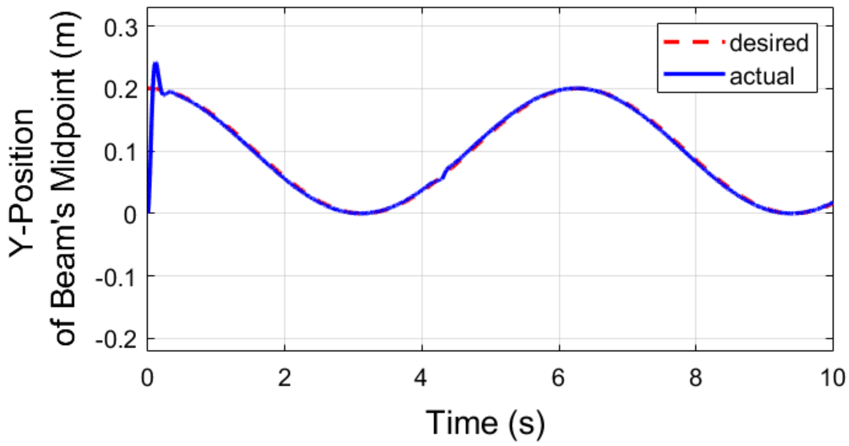

Figure 10. Y-position tracking of the beam, y_o.

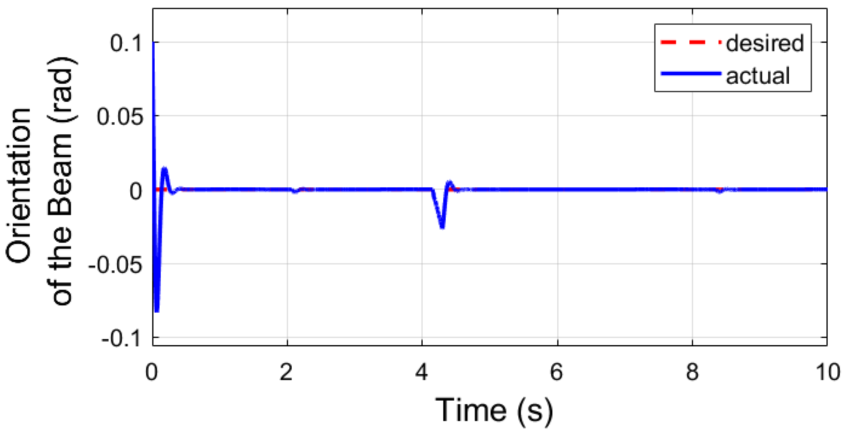

Figure 11. Orientation tracking of the beam, θ.

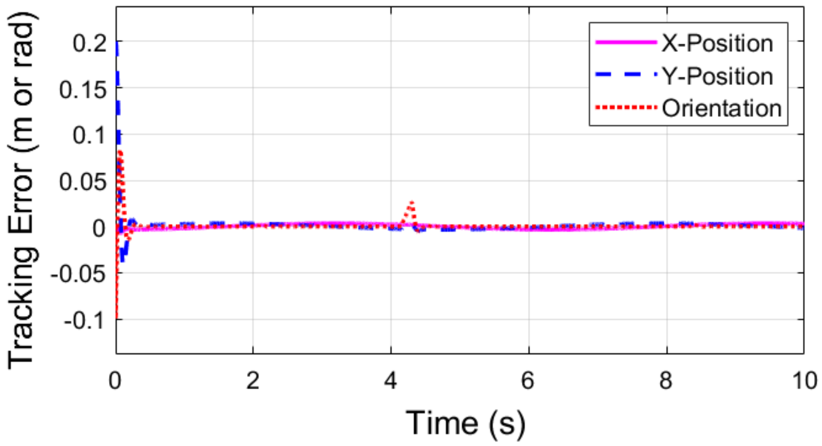

Figure 12. Tracking error, *e* of the beam's midpoint.

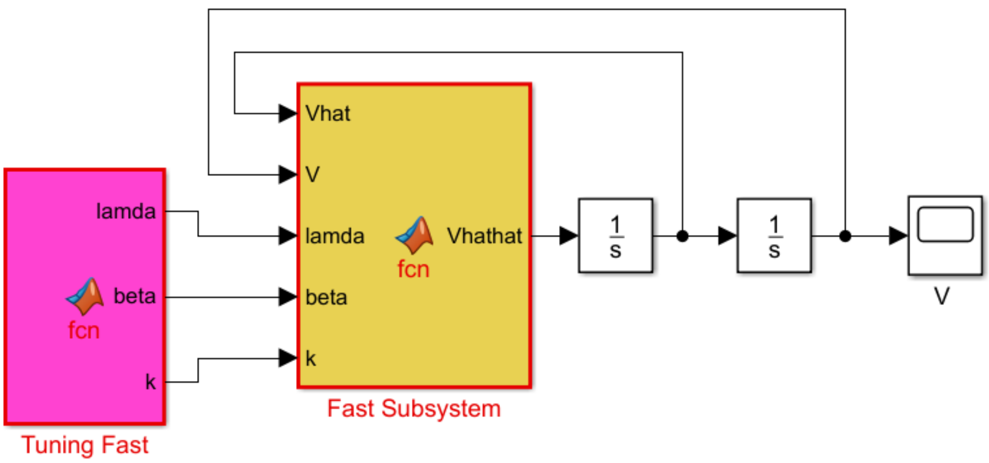

Figure 13. VFC for the fast subsystem in Simulink.

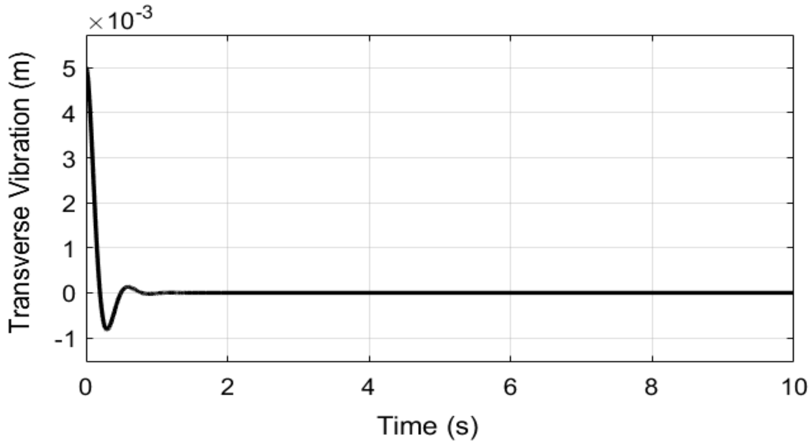

Figure 14. Suppression of the beam's vibration, v_f.

4. CONCLUSION

This paper presents the study on a pair of three-links and planar cooperative manipulators in handling a flexible beam to track desired trajectories and suppress the transverse vibration of the flexible beam. The system was modelled based on PDE-based system and employed the singular perturbation method to produce slow and fast subsystems. A composite control comprising of CTC scheme was designed for a slow subsystem so that the beam's midpoint tracks the desired positions/orientation and VFC was designed for the fast subsystem to suppress the beam's vibration. A stability analysis was carried out for each subsystem to satisfy Tikhonov's theorem. A simulation test was carried out by using the Simulink. For a slow subsystem, the beam's midpoint positions/orientation successfully tracked the desired positions/orientation within 0.5 s with RMSE values of 0.002745 m, 0.02292 m, and 0.01563 rad for x-direction, y-direction and the orientation, respectively. For a fast subsystem, the transverse vibration of the beam was completely suppressed to zero within 0.8 s. The results show the feasibility of the designed control scheme in tracking the desired trajectory while suppressing the vibration of the flexible beam. Future work will focus on the validation of the controller feasibility through experimental hardware tests and new controller design to compensate for uncertainties in the plant.

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