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2-D Analysis Based ILC for Linear Time-Variant Discrete Systems with Multiple Time Delays

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***Abstract*- This paper presents Two-Dimensional (2-D) system analysis based Iterative Learning Control (ILC) methods for Linear Time-Variant (LTV) discrete systems with multiple time delays. Both the state delays and the input delays are considered in the ILC design. The ILC research strategy for LTV discrete systems with multiple time delays is to rescontruct the derived ILC error equations into the 2-D Roessor's model, so that the convergence conditions for the proposed ILC rules can be derived according to the convergence property of the 2-D Roessor's model. A numerical example is used to validate the effectiveness of the proposed ILC methods.**

I. INTRODUCTION

Iterative Learning Control (ILC) is now a well-known control technique for improving the tracking response in systems that repeat a given task or operation over and over again. This makes ILC strategy widely used in the applications of robotic manipulators that are required to repeat a given task with high precision, disk drive systems or more generally, the class of tracking systems [1], [2]. Compared with other conventional control approaches, one of the prominent features of the ILC is that it provides improved performance with reduced knowledge of the plant.

Since the ILC concept was introduced in 1984 by Arimoto *et al*. [1], a lot of research works have been contributed to develop ILC theory with efficient learning algorithms. Good surveys on these existing ILC techniques can be found in [3]-[5]. Among these ILC techniques, it is worth noting that most of them mainly focus on dynamic systems without time delays, and there have been only limited works to study the ILC issue for dynamic systems with time delays [6]-[10]. In many practical applications such as batch processes, and remote controlled robots, and man-machine systems, etc., the existence of time-delays is inherent. A time-delay quite often degrades the performance of a control system, or even destabilizes the whole system. Therefore, the study of ILC for dynamical time-delayed systems has become essential and increasingly important over the past years [6]-[10]. In those limited ILC works to dynamic time-delayed systems, the state delays in uncertain nonlinear continuous-time systems were tackled by a high-order ILC approach in [6]. The initial condition issues on ILC for nonlinear continuous-time systems with state delays were discussed in [7]. The state delays and input delays in linear continuous-time systems were addressed independently by a kind of Two-Dimensional (2-D) system theory based ILC approach in [9]. More recently, by exploiting the theory of 2-D linear inequalities, Li *et al*. [10] further presented an ILC technique to deal with the input delays for nonlinear discrete-time systems. All of these promising ILC works for time-delayed systems, however, treated the state delays and input delays separately.

There exist two different dynamics in an ILC system, namely, the dynamic along the time horizon and the dynamic along the learning iteration. Owing to two independent dynamic directions in 2-D dynamical systems, the 2-D system model provides an excellent mathematical platform to describe both the dynamics along the time horizon and the behavior of the learning iteration in an ILC system. Consequently, 2-D analysis approach has been successfully utilized to design the ILC systems [9]-[14], including the ILC designs for dynamical time-delayed systems [9], [10].

The main objective of this paper is to further extend the existing 2-D system theory based ILC techniques to Linear Time-Variant (LTV) discrete systems with time-delays. Different cases of time delays in LTV discrete systems are investigated. In the study of 2-D analysis based ILC for LTV discrete time-delayed systems, we address the ILC systems with multiple time-delays in state, and the ILC systems with multiple time-delays in input, respectively. Then the derived results are integrated and extended to the case with multiple time-delays both in state and in input. The employed design strategy is to reconstruct the derived ILC error equations in a compact form of the 2-D LTV discrete Roessor's model so that we can present convergent ILC rules based on the property of the 2-D LTV discrete system. An important contribution of this paper is that unlike those existing ILC techniques for time-delayed systems [6]-[10], the state delays and the input delays are considered simultaneously in our ILC design.

This reminder of this paper is organized as follows: Section 2 addresses the 2-D system analysis based ILC techniques for LTV discrete systems with multiple time-delays in state, and Section 3 investigates the same ILC strategy for LTV discrete systems with multiple time-delays in input. Section 2 and Section 3 are synthesized in Section 4. A numerical example is given in Section 5 to demonstrate the effectiveness of the proposed ILC algorithm. Finally, Section 6 presents the conclusions.

II. ILC FOR LTV DISCRETE SYSTEMS WITH MULTIPLE TIME DELAYS IN STATE

Let us first consider the ILC problem of the following LTV discrete system with multiple time delays in state

$$x_k(t+1) = A(t)\cdot x_k(t) + \sum_{i=1}^{p} A_i(t)\, x_k(t-t_i) + B(t)u_k(t), \tag{1a}$$

$$y_k(t) = C(t)\cdot x_k(t), \tag{1b}$$

where k denotes learning iteration, and t is the discrete time index running from 0 to N ($N>0$) to complete a cycle. For all t and k, $x_k(t)\in R^n$, $u_k(t)\in R^m$, and $y_k(t)\in R^p$ are the state vectors, input vectors, and output vectors, respectively. *A(t), B(t), C(t),* and $A_i(t)$ (i = *1,2, … , p*) are real matrices with appropriate dimensions. The delay factors $t_1, t_2, \cdots, t_p$ are assumed with $0 < t_1 < t_2 < \cdots < t_p$. The desired output for system (1) is $y_d(t)\in R^p$, $t = 0,1,2,\cdots,N$.

At each iteration of ILC process, the output tracking error is denoted as follows

$$e_k(t) = y_d(t) - y_k(t). \tag{2}$$

Then, the ILC problem for the LTV discrete system (1) with state delays is stated as follows: Given system (1) with initial state $x_k(t) = w(t)$, $t\in\{-t_p, -t_p+1, \cdots, -2, -1, 0\}$, and the desired output $y_d(t)$, $t = 0,1,2,\cdots,N$, iteratively determine an appropriate control input sequence $\{u_k(t)\}$ at $t = 0,1,2,\cdots,N-1$ such that the ILC tracking error $e_k(t)$ convergences to zero at $t = 1,2,\cdots,N$ as iteration k goes to infinity.

For the ILC of the LTV discrete system (1) with state delays, we adopt the following ILC rule

$$u_{k+1}(t) = u_k(t) + P_k(t+1)\cdot e_k(t+1) \tag{3}$$

at the time step $t = 0,1,2,\cdots,N-1$. In order to prove that the proposed ILC rule (3) can drive the ILC tracking error $e_k(t)$ to zero, the following Lemma 1 will be used.

Lemma 1 [11]. For the Roessor's type model of 2-D LTV discrete system

$$\begin{pmatrix} X(t+1,k) \\ Y(t,k+1)\end{pmatrix} = \begin{pmatrix} A1(t,k) & A2(t,k) \\ A3(t,k) & A4(t,k)\end{pmatrix}\begin{pmatrix} X(t,k) \\ Y(t,k)\end{pmatrix}, \tag{4}$$

where $X(t,k)\in R^{n_1}$, $Y(t,k)\in R^{n_2}$, $A1(t,k)\in R^{n_1\times n_1}$, $A2(t,k)\in R^{n_1\times n_2}$, $A3(t,k)\in R^{n_2\times n_1}$, and $A4(t,k)\in R^{n_2\times n_2}$, boundary conditions for (4) are given by

X(0, k) = 0 for *k* = 0, 1, 2, … and finite *Y(t, 0)* for *t = 0, 1, 2, …* . (5)

If $\rho(A4(t,k)) \le \delta < 1$, $t, k = 0,1,2,\cdots$ ($\rho(.)$ represents the spectral radius of matrix), then, for each t, we have

$$\lim_{k\to\infty}\begin{pmatrix} X(t,k) \\ Y(t,k)\end{pmatrix} = 0.$$

Theorem 1: Consider the ILC for the LTV discrete system (1) with multiple time delays in state. If there exists gain matrix $P_k(t)$ to make the condition

$$\rho(I - C(t)B(t-1)P_k(t)) \le \delta < 1,\ (t = 1,2,\ldots,N) \tag{6}$$

satisfied, then, using the ILC rule (3), we have $\lim_{k\to\infty} e_k(t) = 0$ for $t = 1,2,\cdots,N$.

Proof: Let us denote

$$\eta_k(t) = x_{k+1}(t-1) - x_k(t-1). \tag{7}$$

Using the system (1a) and the ILC rule (3), we have

$$\begin{aligned}\eta_k(t+1) &= x_{k+1}(t) - x_k(t)\\ &= A(t-1)x_{k+1}(t-1)\\ &\quad + \sum_{i=1}^{p} A_i(t-1)x_{k+1}(t-t_i-1) + B(t-1)u_{k+1}(t-1)\\ &\quad - A(t-1)x_k(t-1)\\ &\quad - \sum_{i=1}^{p} A_i(t-1)x_k(t-t_i-1) - B(t-1)u_k(t-1)\\ &= A(t-1)\eta_k(t) + \sum_{i=1}^{p} A_i(t-1)\cdot\eta_k(t-t_i)\\ &\quad + B(t-1)[u_{k+1}(t-1) - u_k(t-1)]\\ &= A(t-1)\eta_k(t)\\ &\quad + \sum_{i=1}^{p} A_i(t-1)\cdot\eta_k(t-t_i) + B(t-1)P_k(t)e_k(t)\end{aligned} \tag{8}$$

Furthermore, let us apply (1b) and (2) to following expression on $e_{k+1}(t)$,

$$\begin{aligned}e_{k+1}(t) &= y_d(t) - y_{k+1}(t)\\ &= [y_d(t) - y_k(t)] - [y_{k+1}(t) - y_k(t)]\\ &= e_k(t) - C(t)\eta_k(t+1)\\ &= e_k(t) - C(t)A(t-1)\eta_k(t)\\ &\quad - \sum_{i=1}^{p} C(t)A_i(t-1)\eta_k(t-t_i)\end{aligned}$$

$$-C(t)B(t-1)P_k(t)e_k(t) \tag{9}$$

Using the following matrix definitions

$$\bar{A}(t)=\begin{bmatrix} A(t-1) & 0 & A_1(t-1) & 0 & \cdots & \vdots & A_P(t-1) \\ I & 0 & 0 & 0 & \cdots & \vdots & 0 \\ 0 & I & 0 & 0 & \cdots & \vdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & I & 0 \end{bmatrix} \tag{10}$$

$$\bar{\theta}_k(t)=\begin{bmatrix} \eta_k(t) \\ \eta_k(t-1) \\ \vdots \\ \eta_k(t-t_1) \\ \vdots \\ \eta_k(t-t_2) \\ \vdots \\ \eta_k(t-t_p) \end{bmatrix};\quad \bar{B}(t)=\begin{bmatrix} B(t-1)P_k(t) \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}; \tag{11}$$

$$\bar{C}(t)=-C(t)\begin{bmatrix} A(t-1) & 0 & A_1(t-1) & 0 & \cdots & A_p(t-1) \end{bmatrix} \tag{12}$$

we can derive the following 2-D LTV discrete Roessor's system based on (8) and (9)

$$\begin{bmatrix} \bar{\theta}_k(t+1) \\ e_{k+1}(t) \end{bmatrix}=\begin{bmatrix} \bar{A}(t) & \bar{B}(t) \\ \bar{C}(t) & I-C(t)B(t-1)P_k(t) \end{bmatrix}\begin{bmatrix} \bar{\theta}_k(t) \\ e_k(t) \end{bmatrix} \tag{13}$$

From the initial condition $x_k(t)=w(t)$, $t\in\{-t_p,-t_p+1,\cdots,-2,-1,0\}$ for ILC system (1) and the definitions of $\eta_k(t)$ and $\bar{\theta}_k(t)$, we know that the boundary condition for 2-D LTV discrete system (13) is $\bar{\theta}_k(1)=0$ for $k=0,1,2,\cdots\cdots$. Theorem 1 is thus proved by applying Lemma 1 to (13). End of the proof.

III. ILC FOR LTV DISCRETE SYSTEMS WITH MULTIPLE TIME DELAYS IN INPUT

Let us then consider the ILC problem of the following LTV discrete system with multiple time delays in input

$$x_k(t+1)=A(t)\cdot x_k(t)+\sum_{j=1}^{q}B_j(t)u_k(t-\tau_j), \tag{14a}$$

$$y_k(t)=C(t)\cdot x_k(t). \tag{14b}$$

The delay factors $\tau_1,\tau_2,\cdots,\tau_q$ in (14) are assumed with $0\le\tau_1<\tau_2<\cdots<\tau_q$. The initial iterative condition is set as $x_k(0)=x_0(0)$ for $k=0,1,2,\cdots\cdots$. As the LTV discrete system (14) iteratively tracks the desired output $y_d(t)$ at $t=1,2,\cdots,N$, we adopt the following ILC rule

$$u_{k+1}(t)=u_k(t)+P_k(t+\tau_1+1)\cdot e_k(t+\tau_1+1) \tag{15}$$

at the time step $t=-\tau_q,-\tau_q+1,-\tau_q+2,\cdots,N-\tau_1-1$, where the definition of $e_k(t)$ in (2) is extended to $e_k(t)=0$ as $t\le 0$, and set $P_k(t)=0$ as $t\le 0$.

Theorem 2: Consider the ILC for the LTV discrete system (14) with multiple time delays in input. If there exists gain matrix $P_k(t)$ to make the condition

$$\rho\left(I-C(t)B_1(t-1)P_k(t)\right)\le\delta<1,(t=1,2,\ldots,N) \tag{16}$$

satisfied, then, using the ILC rule (15), we have $\lim_{k\to\infty}e_k(t)=0$ for $t=1,2,\cdots,N$.

Proof: Using (14), (7) and the ILC rule (15), we obtain for $t=1,2,\cdots,N$,

$$\begin{aligned}\eta_k(t+1) &= x_{k+1}(t)-x_k(t)\\ &= A(t-1)x_{k+1}(t-1)\\ &\quad+\sum_{j=1}^{q}B_j(t-1)u_{k+1}(t-\tau_j-1)\\ &\quad-A(t-1)x_k(t-1)\\ &\quad-\sum_{j=1}^{q}B_j(t-1)u_k(t-\tau_j-1)\\ &= A(t-1)\eta_k(t)\\ &\quad+\sum_{j=1}^{q}B_j(t-1)\cdot\left[u_{k+1}(t-\tau_j-1)-u_k(t-\tau_j-1)\right]\\ &= A(t-1)\eta_k(t)\\ &\quad+\sum_{j=1}^{q}B_j(t-1)\cdot P_k(t-\tau_j+\tau_1)e_k(t-\tau_j+\tau_1)\end{aligned} \tag{17}$$

$$\begin{aligned}e_{k+1}(t) &= y_d(t)-y_{k+1}(t)\\ &=\left[y_d(t)-y_k(t)\right]-\left[y_{k+1}(t)-y_k(t)\right]\\ &= e_k(t)-C(t)\eta_k(t+1)\end{aligned}$$

$$= e_k(t) - C(t)A(t-1)\eta_k(t) - \sum_{j=1}^{q} C(t)B_j(t-1)P_k(t-\tau_j+\tau_1)e_k(t-\tau_j+\tau_1) \quad (18)$$

Let us denote the following matrices

$$\tilde{A}(t) = \begin{bmatrix} A(t-1) & 0 & \cdots & B_2(t-1)P_k(t-\tau_2+\tau_1) & 0 & \cdots & B_3(t-1)P_k(t-\tau_3+\tau_1) & 0 & \cdots & \vdots & B_q(t-1)P_k(t-\tau_q+\tau_1) \\ 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & \vdots & 0 \\ 0 & I & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & \vdots & 0 \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & I & 0 \end{bmatrix}$$

$$\tilde{\theta}_k(t) = \begin{bmatrix} \eta_k(t) \\ e_k(t-1) \\ e_k(t-2) \\ \vdots \\ e_k(t-\tau_q+\tau_1) \end{bmatrix}; \qquad \tilde{B}(t) = \begin{bmatrix} B_1(t-1)P_k(t) \\ I \\ 0 \\ \vdots \\ 0 \end{bmatrix};$$

$$\tilde{C}(t) = -C(t)\left[A(t-1) \quad 0 \quad \cdots \quad B_2(t-1)P_k(t-\tau_2+\tau_1) \quad 0 \quad \cdots \quad B_3(t-1)P_k(t-\tau_3+\tau_1) \quad 0 \quad \cdots \quad \cdots \quad B_q(t-1)P_k(t-\tau_q+\tau_1)\right]$$

then, we derive the following 2-D LTV discrete Roessor's system based on (17) and (18)

$$\begin{bmatrix} \tilde{\theta}_k(t+1) \\ e_{k+1}(t) \end{bmatrix} = \begin{bmatrix} \tilde{A}(t) & \tilde{B}(t) \\ \tilde{C}(t) & I - C(t)B_1(t-1)P_k(t) \end{bmatrix} \begin{bmatrix} \tilde{\theta}_k(t) \\ e_k(t) \end{bmatrix} \quad (19)$$

From the initial iterative condition $x_k(0) = x_0(0)$ for $k = 0,1,2,\cdots,p$ the extended definition $e_k(t) = 0$ for $t \le 0$, and the definitions of $\eta_k(t)$ and $\tilde{\theta}_k(t)$, we know that the boundary condition for 2-D LTV discrete system (19) is $\tilde{\theta}_k(1) = 0$ for $k = 0,1,2,\cdots,p$. Theorem 2 is thus proved by applying Lemma 1 to (19). End of the proof.

IV. ILC FOR LTV DISCRETE SYSTEMS WITH MULTIPLE TIME DELAYS BOTH IN STATE AND IN INPUT

In this section, Section 2 and Section 3 will be integrated to address a more complicated case. For the ILC problem of the following LTV discrete system (20), the time delays both in state and in input will be investigated together

$$x_k(t+1) = A(t)\cdot x_k(t) + \sum_{i=1}^{p} A_i(t)x_k(t-t_i) + \sum_{j=1}^{q} B_j(t)u_k(t-\tau_j) \quad (20a)$$

$$y_k(t) = C(t)\cdot x_k(t). \quad (20b)$$

The delay factors $t_1, t_2, \cdots, t_p$ and $\tau_1, \tau_2, \cdots, \tau_q$ are assumed with $0 < t_1 < t_2 < \cdots < t_p$ and $0 \le \tau_1 < \tau_2 < \cdots < \tau_q$. The initial iterative condition is set as $x_k(t) = w(t)$ for $t \in \{-t_p, -t_p+1, \cdots, -2, -1, 0\}$ and $k = 0,1,2,\cdots$. As the LTV discrete system (20) iteratively tracks the desired output $y_d(t)$ at $t = 1,2,\cdots,N$, we adopt the following ILC rule

$$u_{k+1}(t) = u_k(t) + P_k(t+\tau_1+1)\cdot e_k(t+\tau_1+1) \quad (21)$$

at the time step $t = -\tau_q, -\tau_q+1, -\tau_q+2, \cdots, N-\tau_1-1$, where the definition of $e_k(t)$ in (2) is extended to $e_k(t) = 0$ as $t \le 0$, and set $P_k(t) = 0$ as $t \le 0$.

Theorem 3: Consider the ILC for the LTV discrete system (20) with multiple time delays both in state and in input. If there exists gain matrix $P_k(t)$ to make the condition

$$\rho\left(I - C(t)B_1(t-1)P_k(t)\right) \le \delta < 1, (t = 1,2,\ldots,N) \quad (22)$$

satisfied, then, using the ILC rule (21), we have $\lim_{k\to\infty} e_k(t) = 0$ for $t = 1,2,\cdots,N$.

Proof: Using (20), (7) and the ILC rule (21), we obtain for $t = 1, 2, \cdots, N$,

$$\eta_k(t+1) = x_{k+1}(t) - x_k(t)$$

$$
\begin{aligned}
&= A(t-1)x_{k+1}(t-1) + \sum_{i=1}^{p} A_i(t-1)x_{k+1}(t-t_i-1) \\
&\quad + \sum_{j=1}^{q} B_j(t-1)u_{k+1}(t-\tau_j-1) \\
&\quad - A(t-1)x_k(t-1) - \sum_{i=1}^{p} A_i(t-1)x_k(t-t_i-1) \\
&\quad - \sum_{j=1}^{q} B_j(t-1)u_k(t-\tau_j-1) \\
&= A(t-1)\eta_k(t) + \sum_{i=1}^{p} A_i(t-1)\eta_k(t-t_i) \\
&\quad + \sum_{j=1}^{q} B_j(t-1)\cdot\left[u_{k+1}(t-\tau_j-1) - u_k(t-\tau_j-1)\right]
\end{aligned}
$$

$$
\begin{aligned}
&= A(t-1)\eta_k(t) + \sum_{i=1}^{p} A_i(t-1)\eta_k(t-t_i) \\
&\quad + \sum_{j=1}^{q} B_j(t-1)\cdot P_k(t-\tau_j+\tau_1)e_k(t-\tau_j+\tau_1)
\end{aligned}
\tag{23}
$$

$$
\begin{aligned}
e_{k+1}(t) &= y_d(t) - y_{k+1}(t) \\
&= \left[y_d(t) - y_k(t)\right] - \left[y_{k+1}(t) - y_k(t)\right] \\
&= e_k(t) - C(t)\eta_k(t+1) \\
&= e_k(t) - C(t)A(t-1)\eta_k(t) \\
&\quad - \sum_{i=1}^{p} C(t)A_i(t-1)\eta_k(t-t_i) \\
&\quad - \sum_{j=1}^{q} C(t)B_j(t-1)P_k(t-\tau_j+\tau_1)e_k(t-\tau_j+\tau_1)
\end{aligned}
\tag{24}
$$

Let us define the following matrices

$$
M(t) = \begin{bmatrix}
0 & \cdots & B_2(t-1)P_k(t-\tau_2+\tau_1) & 0 & \cdots & B_3(t-1)P_k(t-\tau_3+\tau_1) & 0 & \cdots & \vdots & B_q(t-1)P_k(t-\tau_q+\tau_1) \\
0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & \vdots & 0 \\
I & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & \vdots & 0 \\
0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & \vdots & 0 \\
\vdots & \cdots & \vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots & \vdots \\
\vdots & \cdots & \vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots & \vdots \\
0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & I & 0
\end{bmatrix}
$$

$$
N(t) = \begin{bmatrix}
0 & 0 & \cdots & 0 & \cdots & 0 & 0 \\
I & 0 & \cdots & 0 & \cdots & 0 & 0 \\
0 & I & \cdots & 0 & \cdots & 0 & 0 \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
0 & 0 & \cdots & 0 & \cdots & I & 0
\end{bmatrix};\;
V(t) = \begin{bmatrix} B_1(t-1)P_k(t) \\ 0 \\ \vdots \\ 0 \end{bmatrix};\;
E(t) = \begin{bmatrix} \overline{A}(t) & M(t) \\ 0 & N(t) \end{bmatrix};
$$

$$
\theta_k(t) = \begin{bmatrix} \overline{\theta}_k(t) \\ e_k(t-1) \\ e_k(t-2) \\ \vdots \\ e_k(t-\tau_q+\tau_1) \end{bmatrix};\quad
F(t) = \begin{bmatrix} V(t) \\ I \\ 0 \\ \vdots \\ 0 \end{bmatrix};\quad
G(t) = \left[\overline{C}(t) \quad H(t)\right].
$$

$$
H(t) = -C(t)\left[0 \quad \cdots \quad B_2(t-1)P_k(t-\tau_2+\tau_1) \quad 0 \quad \cdots \quad B_3(t-1)P_k(t-\tau_3+\tau_1) \quad 0 \quad \cdots \quad \cdots \quad B_q(t-1)P_k(t-\tau_q+\tau_1)\right]
$$

where $\overline{A}(t)$, $\overline{C}(t)$ and $\overline{\theta}_k(t)$ are defined as in (10), (11), and (12), respectively.

Using the above matrix definitions, we can derive the following 2-D LTV discrete Roessor's system based on (23) and (24)

$$\begin{bmatrix} \theta_k(t+1) \\ e_{k+1}(t) \end{bmatrix} = \begin{bmatrix} E(t) & F(t) \\ G(t) & I - C(t)B_1(t-1)P_k(t) \end{bmatrix} \begin{bmatrix} \theta_k(t) \\ e_k(t) \end{bmatrix} \tag{25}$$

From the initial iterative condition $x_k(t) = w(t)$ for $t \in \{-t_p, -t_p+1, \cdots, -2, -1, 0\}$ and $k = 0, 1, 2, \cdots, p$, the extended definition $e_k(t) = 0$ for $t \le 0$, and the definitions of $\eta_k(t)$ in (7), $\overline{\theta}_k(t)$ in (11), and $\theta_k(t)$, we know that the boundary condition for 2-D LTV discrete system (25) is $\theta_k(1) = 0$ for $k = 0, 1, 2, \cdots, p$. Theorem 3 is thus proved by applying Lemma 1 to (25). End of the proof.

From Theorem 3 and the ILC rule (21), we know that for the LTV discrete system (20), there are multiple time delays included both in state and in input, but the proposed ILC rule (21) is only related to the smallest time delay factor in input. This point is similar to the results obtained in [10], but the state delays aren't included there. It can be further illustrated by the simulation in Section 5.

V. Simulation

Example: Consider an ILC problem of the following LTV discrete system with time-delays both in state and in input

$$x_k(t+1) = A(t)x_k(t) + A_1(t)x_k(t-3) + B_1(t)u_k(t-2) + B_2(t)u_k(t-4) \tag{26a}$$

$$y_k(t) = C(t)x_k(t), \quad t \in \{0, 1, 2, \cdots\cdots, 100\} \tag{26b}$$

where $x_k(t) = \begin{bmatrix} x_k^{(1)}(t) & x_k^{(2)}(t) \end{bmatrix}^T$,

$$A(t) = \begin{bmatrix} -0.25 & -0.22\cos t \\ -0.46 & 0.3\sin t - 0.05 \end{bmatrix},$$

$$A_1(t) = \begin{bmatrix} 0.01t & -0.06 \\ 0.15 & 0.5\sin(2t) \end{bmatrix}, B_1(t) = \begin{bmatrix} 0.027t+1 \\ 0.12 \end{bmatrix},$$

$B_2(t) = \begin{bmatrix} -0.4 \\ 0.8\cos t \end{bmatrix}$, and $C(t) = \begin{bmatrix} 0.45 & -0.001t \end{bmatrix}$. The initial value of state variable is $x_k(t) = \begin{bmatrix} 0 & 0 \end{bmatrix}^T$ for t = -3, -2, -1, 0. The desired output $y_d(t)$ is described as

$$y_d(t) = 1.2\sin(0.05t) + 0.5, \quad t \in \{0, 1, 2, \cdots\cdots, 100\}.$$

Regarding the iterative initial condition, it is noticed that we don't require $y_d(0) = y_k(0)$. The accuracy of ILC is evaluated by the following total square error of tracking

$$EE_k = \sum_{t=1}^{100} \left[y_d(t) - y_k(t) \right]^2.$$

In the ILC process of system (26), provided that we have no accurate information on matrices *A(t), A*$_1$*(t), B*$_1$*(t), B*$_2$*(t),* and *C(t)*. The estimated values for matrices *B*$_1$*(t)* and *C(t)* are given as

$$\hat{B}_1(t) = \begin{bmatrix} 0.03(t-1)+1.2 \\ 0.18 \end{bmatrix} \quad \text{and} \quad \hat{C}(t) = \begin{bmatrix} 0.4 & -0.003t \end{bmatrix}.$$

Regarding the delay factors in (26), we only know the accurate value of the smallest delay factor τ_1 in input as $\tau_1 = 2$.

As the proposed ILC rule (21) for the LTV discrete system (20) is used, we set the initial control input as $u_0(t) = \begin{bmatrix} 0 & 0 \end{bmatrix}^T$ for $t \in \{-4, -3, -2, \cdots\cdots, 96, 97\}$. The gain matrix $P_k(t)$ is selected as

$$P_k(t) = 0.5\left(\hat{C}(t)\hat{B}_1(t-1)\right)^T \left[\hat{C}(t)\hat{B}_1(t-1)\left(\hat{C}(t)\hat{B}_1(t-1)\right)^T \right]^{-1}$$

for $t \in \{1, 2, \cdots\cdots, 100\}$, and $P_k(t) = 0$ for $t \le 0$. Obviously, the determined gain matrix $P_k(t)$ can make

$$\rho\left(I - C(t)B_1(t-1)P_k(t)\right) \le \delta < 1$$

for $t \in \{1, 2, \cdots\cdots, 100\}$. Fig. 1 shows the curve of the total squared error EE_k of tracking in the process of ILC rule (21) being iteratively executed.

When the ILC rule (21) is iteratively executed for 5 and 6 times, respectively, the tracking performance of the ILC system (26) to the desired output $y_d(t)$ is illustrated in Fig. 2. And Fig. 3 presents the resultant control input after the ILC system (26) convergences using the ILC rule (21). From Fig. 1 and Fig. 2, it can be noticed that the convergence rate of the proposed ILC rule (21) is high and the ILC system output $y_k(t)$ is capable of approaching the desired trajectory $y_d(t)$ accurately within few iterations.

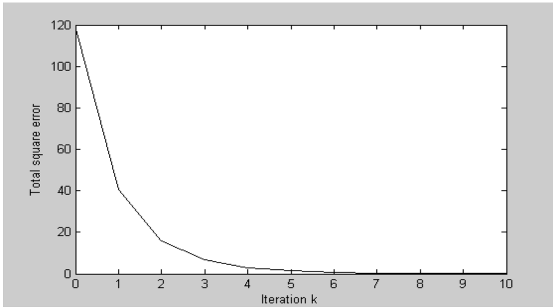

Fig. 1. The curve of the total squared error EE_k of tracking in the process of ILC rule (21) being iteratively executed

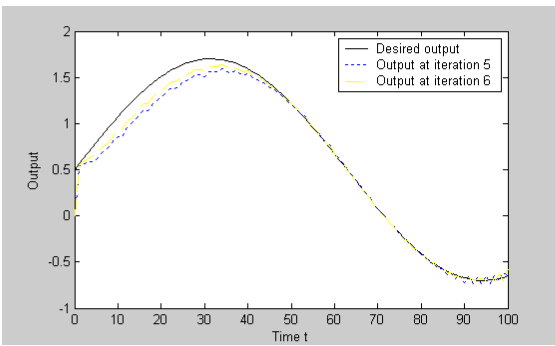

Fig. 2. The tracking performance of the ILC system (26) to the desired output $y_d(t)$ when the ILC rule (21) is iteratively executed for 5 and 6 times, respectively

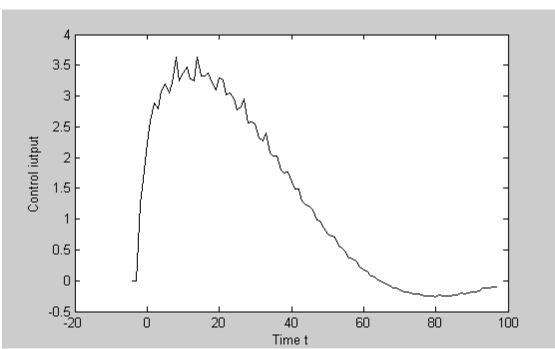

Fig. 3. The resultant control input after the ILC system (26) convergences using the ILC rule (21)

VI. CONCLUSION

We have shown that the 2-D LTV discrete Roesser's model can be applied to describe the ILC process of LTV discrete systems with multiple time delays. As a result, ILC rules with convergent conditions can be derived based on the convergence property of the 2-D LTV discrete Roesser's model. Different situations of time delays in LTV discrete systems are discussed. From both theoretical investigation and simulation, it can be concluded that there are multiple time delays included in LTV discrete systems, but the proposed ILC rule is only related to the smallest delay factor in input.

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